STATI – СТАТЬИ – ARTICLES – AUFSÄTZE

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GRAPHS AND OPPOSITIONS

0 In the present paper we will try to demonstrate some of advantages of considering a phonemic system as a graph, although the paper is only a short sketch of the whole problem.

The graphic expression of phonemic systems is a common practice in phonology since Trubetzkoy, though such a practice is not based on the graph theory, but more on the intuition of scholars.

1 Opposition and phonemic system

The set of phonemes is considered finite. This set is an unordered set, because the substitution test has not revealed relations between phonemes. That which makes this set to be a phonemic system is some describable structure over the set of phonemes.

Note 1: All observances could be related not only to phonemic systems, but could also be applied on all equivalent systems revealed by linguistic analysis, especially on morphemic systems.

The phonemic system is then, on the one hand, a set of phonemes, and a structure over the set on the other. Individual phonemic systems thus differ not only by different sets, but also by different structures.

Note 2: The sets of carbon atoms are the same for diamond and for graphite, but the difference between both is carried out just by the structure.

The structure of a phonemic system is given by oppositions. The set of all oppositions in the given system we will mark as O.

Opposition is a relation between elements of a given set (here a set of phonemes). Considering the set of phonemes *P*, then every describable relation between phonemes (i.e. elements of the set *P*) is an opposition.

Note 3: For the definition of phonemic oppositions from the point of view of the Prague School, which could be considered as classical, see Trubetzkoy 1939: 30-31

The concept of opposition could be made more specified and this will be attempted in the following lines.

Phonemic system is defined as an ordered pair */P, O/*.

Phonemic subsystem */P´, O´/* is defined as a proper subset of the phonemic system */P, O/*. All properties of the phonemic system listed below are valid for subsystems, too.

Note 4: The preceding paragraph does not claim that a subsystem of a given system has necessarily the same properties as its superior system!

Note 5: For the sake of illustration and out of practical reasons, only several selected subsystems of phonemic systems will be shown in the examples below.

2 Oppositions

Oppositions as defined above exist between all elements of a given phonemic system, i.e. there is an opposition between any pair of phonemes.

Besides, each phoneme is in **null** opposition with itself (*/x/* → */x/*).

If there is an opposition between the phoneme */x/* and the phoneme */y/*, then there necessarily exists an opposition between the phoneme */y/* and the phoneme */x/* (i.e. if the opposition */x/* → */y/* exists, then */y/* → */x/* exists, too). The first opposition will then be arbitrarily termed **oriented**, the second is oriented against the first (**inversed oriented**).

Note 6: On orientation of oppositions see below.

We will call such a defined opposition **general opposition** (symbolically as O^G).

The phonemic system /*P*, O^G/ is then termed as **gross phonemic system**.

General oppositions distinguish phonemes from one another (or in other words, elements of a set of elements of the set *P* are distinguishable thanks to general oppositions). However, a detailed description of relations between phonemes is not possible, because triviality of general oppositions does not allow for any specification of the opposition.

Be it as it may, any general opposition (with an exception of null oppositions) between two phonemes is accompanied with an inversed oriented opposition.

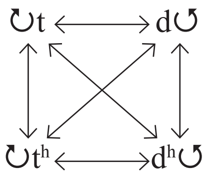
The number of null oppositions in a given gross phonemic system is equal to the number of phonemes of the given system (= *n)*.

The number of non-null general oppositions in a given gross phonemic system is given by the formula *n(n-1)/2*, where *n* is the number of phonemes of the given system.

The total number of all oppositions in a given system, including null oppositions, is then given by the formula: *n(n-1)/2 + n*.

Note 7: In the following lines we will mark for simplicity's sake two contrary oriented oppositions between a pair of phonemes not by two arrows, but by only one two-headed arrow. Any null opposition is then marked by the loop.

Example 1: In the gross subsystem of the Vedic alveolar stops (*/t/*, */d/*, /t^h/, /d^h/) we can see four null oppositions, twelve non-null oppositions; in total sixteen general oppositions.



Considering the specification of the oppositions, towards a more detailed description of relations between phonemes, we can introduce the term **set of** (phonemic) **values** (*V/x/*), or, from this notion derived, the term phonemic **feature**, respectively (for a more detailed description of the term set of values see Marcus 1967: 48-9; Marcus 1969: 51-3; Šefčík 2008: 5-7; Šefčík 2009: 186-7).

For the purposes of this paper it is sufficient that there is a bijective relation between every single phoneme and exactly one set of values and, vice versa, every phoneme is uniquely identified by its set of values.

Any pair of values which are mutually contrastive, homogeneous and incompatible constitutes the values of a single feature (Šefčík 2008:7; Šefčík 2009: 189-187). One of the values is arbitrarily chosen as unmarked and hence given the value *0*, the second as marked and and given the value *1*. Comparing both pairs of sets of values we can determine in how many features both phonemes differ.

In such an approach to the features, any set of values of phoneme is then expressible as an ordered linear sequence of *0*s a *1*s, or in other words, as a code. Differences between codes are known as so-called **Hamming distance** (first published in Hamming 1950). For the sake of simplicity, Hamming distances between sets of values of phonemes will be marked as *d(/x/, y/)*.

The most interesting oppositions are those between phonemes with the least possible difference between their sets of values, i.e. such oppositions for which the difference between sets of values *V/x/* and *V/y/* is equal to 1, and hence the Hamming distance between phonemes equal to 1. Such oppositions will be called here **minimal oppositions** and we will mark them symbolically as O^F. The phonemic system with only minimal oppositions will be given a name **fine phonemic system**.

Any gross opposition (including null oppositions) is then expressed as a succession of minimal oppositions.

Another way to describe minimal oppositions is to define them as such oppositions between elements of the system (i.e. phonemes) which could not be divided to other oppositions on a given level of analysis (cf. Hjelmslev 1963: def. 1, 2, 3 8, 44).

Obviously, any fine phonemic system is always a subsystem of a given gross phonemic system.

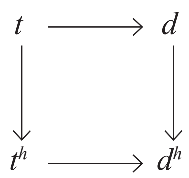
3 Graphic expression of phonemic system

We can illustrate any phonemic system, either gross or fine, using the graph. In the graph the phonemes will be expressed as vertices (points, nodes) of the graphs, oppositions drawn as edges (lines or arrows for oriented graphs) (cf. Sedláček 1977: 26, 146; Diestel 2000: 2, 25; Demel 2002: 11-15).

In the graph we can favorably replace edges with arrows in order to express orientation of the opposition in the following manner: the head of the arrow is placed next to the marked member of an opposition for any minimal opposition. For general oppositions the arrows are arbitrarily; we must only keep in mind that the same number of arrows pointing out from any vertex should be pointing to it.

Null oppositions are always marked (if it is necessary to mark them) as loops, coming out and striking on the same vertex.

Example 2: The subsystem of minimal oppositions between Vedic phonemes /*t*/, /*d*/, /*t*^{h}/, /*d*^{h}/ could be expressed using orientation of the graph in the following way (the features used are /-voiced/ → /+voiced/ and /-aspirated/ → /+aspirated/):

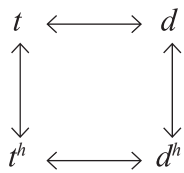


4 Symmetricity of the system

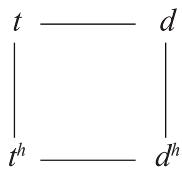
In some cases it is not necessary to work with the orientation of oppositions. If the orientation is not necessary, we can neglect it in the following way. Every minimal opposition can be replaced by a gross opposition (or in other words, to every oriented opposition we will add an inverse oriented opposition). Such process can be termed **symmetrization** of the system. The arrows could then be omitted at all (cf. Demel 2002: 14).

Note 8: Null oppositions are symmetric by their very nature; thus they can be left out, provided they are not necessary.

Example 3: The formal example 2 could be symmetrized as:



Example 4: The same subsystem symmetrized without marking of orientation:



In the graphs expressing fine phonemic (sub)systems the smallest number of the edges between two phonemes is equal to their minimal Hamming distance.

Example 5: In the above mentioned subsystem of minimal oppositions between Vedic /*t*/, /*d*/, /t^h/, /d^h/ it holds that d(/*t*/, /*d*/) = d(/t^h/,/d^h/) = d(/*t*/,/t^h/) = d(/*d*/,/d^h/) = 1, and d(/*t*/,/d^h/) = d(/*d*/,/t^h/) = 2.

5 Completeness of the system

Any system is **complete** if there exists an opposition between any pairs of phonemes of a given (sub)system. If the (sub)system is not complete, we call it **incomplete** (cf. Sedláček 1977: 27; Diestel 2000: 3; Demel 2002: 21).

Hence, the gross phonemic (sub)system is always complete.

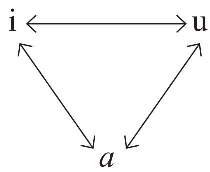
On the contrary, a fine phonemic system could be both either complete or incomplete.

The subsystem of a complete fine phonemic system is always complete; the subsystem of an incomplete fine phonemic system could be both either complete or incomplete.

Example 6: The above mentioned example 1 illustrates a complete gross phonemic subsystem.

Example 7: The above mentioned examples 2, 3, 4 illustrate an incomplete fine phonemic subsystem.

Example 8: The subsystem of Vedic vowels is a complete fine phonemic subsystem:



6 The path in the phonemic system

We will term as **path** such unrepeating sequences of minimal oppositions (expressed by edges) leading from one phoneme to another (without loops, i.e. null oppositions).

The most important are **minimal paths**, i.e. the shortest of possible paths, or in other words, such a path formed by the smallest total number of edges (minimal oppositions) between phonemes. The length of the minimal path is equal to the Hamming distance between both phonemes (cf. Sedláček 1977: 41; Diestel 2000: 6-8; Demel 2002: 20).

Example 9: The Vedic phonemes /*t*/ and /*d*/ differ only in the feature /±voice/, hence their mutual Hamming distance is equal to 1. The minimal path between phonemes is equal to only one edge.

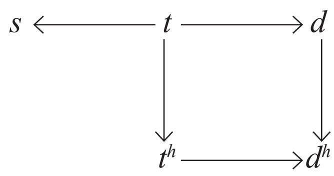
Example 10: The Vedic phonemes /*t*/ and /d^h/ differ only in the features /±voiced/ and /±aspiraed/. Hence the Hamming distance between both phonemes is equal to 2 and the minimal path between both phonemes is equal to 2, too. There are two ways how to draw the minimal path, one /*t*/ → /*d*/ → /d^h/, the second /*t*/ → /t^h/ → /d^h/!

Example 11: The minimal path between the phoneme /*t*/ and itself is equal to 0, as the phoneme is in null opposition with itself.

Example 12: The path between Vedic phonemes /*t*/ – /*d*/ – /d^h/ – /t^h/ is not minimal, as there is a shorter path /*t*/ – /t^h/ with the length equal to 1.

Example 13: The path between the Vedic phonemes /*d*/ – /*t*/ – /*s*/ is a minimal path of the length equal to 2, as there is no shorter path between the first and the third phoneme.

Note 9: The picture given below illustrates the examples 9–12.



7 Final remarks

In the preceding lines we tried to demonstrate that it may be possible to interpret phonemic systems with the use graph theory. However, more was left unresolved rather than revealed due to limitations of such a simple introduction to this field of study as this paper meant to be.

Graph theory offers a formal method and formal instruments for describing any systems, including phonemic. The properties of systems, as generally described by graph theory, could hence be applied on phonemic systems. Some of such properties and their applications were demonstrated above.

The more formal approach to the study of phonemic system is always welcomed and it is graph theory that offers such a formal approach.

LITERATURE

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GRAFY A OPOZICE

V článku se hovoří o možnosti využít teorii grafu pro popis fonologických systémů a uvádění se některé paralely ve vlastnostech fonologických systémů a grafů a možnosti aplikací teorie grafů na popis fonologických systémů. Ukazují se rozdíly mezi tzv. hrubým fonologickým systémem a tzv. jemným fonologickým systémem, neboť u prvního typu fonologického systému nerozlišujeme mezi opozicemi (pracujeme se všemi možnými protiklady mezi prvky daného systému), zatímco u druhého typu fonologického systému pracujeme jen s opozicemi minimálními, tj. takovými, které existují jen mezi fonémy s nejmenšími možnými rozdíly mezi svými množinami hodnot (distinktivních rysů). Opozice obou typů je možné v grafu vyjádřit hranami, zatímco fonémy jako vrcholy daných grafů. V článku se také probírají některé možné vlastnosti, které je u takto pojatých systémů vyšetřovat: symetričnost a kompletnost systému, orientaci opozic a délku nejkratší možné cesty mezi fonémy daného systému, která vyjadřuje graficky vzdálenost mezi množinami vlastností daných fonémů, respektive mezi fonémy samotnými.

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