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A CASE OF TEACHER AND STUDENT MATHEMATICAL PROBLEM-SOLVING BEHAVIORS FROM THE PERSPECTIVE OF COGNITIVE-METACOGNITIVE FRAMEWORK

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Abstract

This study aims to investigate the problem-solving behaviors of a teacher and his students based on a cognitive–metacognitive framework. The problem-solving behaviors of 6–8th-grade students and a mathematics teacher were recorded and encoded during task-based interview sessions about solving problems, and semi-structured interviews were used to obtain information regarding the mathematics teacher's perceptions of mathematical problem-solving processes. They solved the problems in a learning environment, and their problem-solving processes were investigated using the think-aloud method. The results indicated that the students and the teacher followed a similar path involving reading, understanding, exploring, planning, and implementing. Furthermore, not all episodes occurred in each problem-solving task and the behaviors that represented given episodes changed according to the participants. Students with different problem-solving skill levels were found to exhibit different frequencies of cognitive and metacognitive behaviors while solving problems. The problem-solving behaviors of the teacher and the students revealed information related to metacognitive behaviors that are to be developed in further studies.

Keywords

metacognitive behaviors, cognitive behaviors, mathematics education, problem-solving

Introduction

Learning to argue has been considered by Krummheuer (2007) a *sophisticated mathematical level* that a mathematics learning environment should aim for. This process embraces the collective activities of a specific group and the individual contributions of that group's members (Krummheuer, 2007). In other words, argumentation is home to individual practices along with social practices (Kuhn, Zillmer, Crowell, & Zavala, 2013), which reinforces justification and actively monitoring the individual's reasoning and others' thinking procedures, which redirects us to problem-solving processes (Kuhn, 1991). It is also possible to say that argumentation skills and individual problem-solving skills are transferable to one another (Cho & Jonassen, 2002). Therefore, argumentation and problem-solving are *higher-order thinking* processes that rely on some similar cognitive and metacognitive attempts (Ku & Ho, 2010). It is possible to say that development in one of these thinking skills may imply development in the other.

Problem-solving is at the heart of argumentation and one of the significant topics involved in all levels of education from kindergarten to teacher training programs. Problem-solving in mathematics curricula is frequently associated with certain behaviors, including decision-making related to procedural operations, mathematical reasoning, organization of knowledge, and monitoring one's own work (Department for Education, 2000; Ministry of National Education, 2013; National Council of Teachers of Mathematics [NCTM], 2000). These behaviors are called managerial decisions or metacognitive skills that are required for problem-solving processes (Artzt & Armour-Thomas, 1992; Flavell, 1976; Garofalo & Lester, 1985; Mayer, 1998; Schoenfeld, 1985). The influences of metacognition on problem-solving has been signified in some studies; learners possessing metacognitive skills performed better in skills related to problem-solving (García, Rodríguez, González-Castro, González-Pienda, & Torrance, 2016; NCTM, 2000; Özsoy, 2007). Mathematical problem-solving requires the ability to select and apply appropriate cognitive strategies for the given task to understand, represent, and solve the problem as well as self-awareness of performance. Thus, it lies at the intersection of cognitive and metacognitive strategies and processes and therefore depends on both cognitive and metacognitive processes (Mayer, 1998; Montague & Applegate, 1993). Additionally, studies have revealed a relationship between achievement in mathematics and metacognitive skills (Özsoy, 2011; Schneider & Artelt, 2010). International examinations also reflect this perspective; for example, the OECD Programme for International Student Assessment (PISA, 2013) encourages the development of metacognition skills in students through problem-solving (Welsh Government, 2012).

Metacognitive strategies may change across age, experience, and self-perceptions of capabilities and limitations (Flavell, 1976). Going beyond individual effort, Schoenfeld (1992) stated that mathematical thinking and understanding were socially constructed and socially transmitted through experiences with mathematics. Similarly, metacognition in general can be developed employing instructional practices (Ader, 2013; Desoete, 2007; Papaleontiou-Louca, 2003). Research has provided several metacognitive training programs for learners to develop their metacognitive knowledge and skills (Baten, Praet, & Desoete, 2017; Schraw, 1998). In mathematics teacher training programs, teachers have been provided information related to mathematical problem-solving processes, cognitive and metacognitive knowledge, skills, and strategies in order to transfer them into their teacher practices and so to their students. It is still a question for further research to understand the behaviors of teachers trained in such a tradition and their students' problem-solving behaviors in terms of the extent to which such skills and knowledge has been transferred from the teachers to the students. Knowing the commonalities and differences in the problem-solving practices of such students and teachers may provide specific information related to the students' performance in mathematics, as Desoete and De Craene (2019) recommended for further study. Before describing the present research, we will first describe problem-solving in terms of metacognition and the place of the cognitive–metacognitive framework within metacognition studies in detail in order to provide information about the nature of the study. Then, we will explain the purpose of the study before presenting information relating to the participants, data, cognitive–metacognitive framework analysis, and results. Finally, we will discuss the results in terms of the performance of the teacher and the students while solving tasks.

Metacognition and Problem-Solving

The concept of metacognition came to light by the end of the 1970s with studies on the constructs of problem-solving rules, the structure of memory, and knowledge of representations within the "architecture of cognition" (Schoenfeld, 1992). The procedure of metacognitive thinking has become the primary focus of many researchers (Artz & Armour-Thomas, 1992, 2001; Flavell, 1976; Garofalo & Lester, 1985; Hartman & Sternberg, 1993). Flavell defined metacognition as "one's knowledge concerning one's own cognitive processes and products or anything related to them" (Flavell, 1976, p. 232), and this definition became the origin of further studies about metacognition.

The difference between cognition and metacognition was identified as cognition invoking a cognitive process and metacognition monitoring that process (Flavell, 1976). In a writing process, for example, the decision to write is deemed to be metacognitive, while the writing activity is considered cognitive (Jacobse & Harskamp, 2012). Garofalo and Lester (1985) stated that "cognition is involved in doing whereas metacognition is involved in choosing and planning what to do and monitoring what is being done" (Garofalo & Lester, 1985, p. 164) and divided metacognition into two parts: knowledge of cognition and regulation of cognition. Knowledge of cognition is similar to the "metacognitive knowledge" definition from Flavell (1979) and is formed by the influence of person (knowledge of one's own capabilities and limitations), task (knowledge of what makes a task more difficult or beliefs about the nature of the task), and strategy (knowing where, when, and how to apply it for a certain task) factors on performance. Metacognitive strategies have been mentioned with metacognitive monitoring and control (Flavell, 1976; Özsoy, 2007; Schoenfeld, 1992), and a problem-solving task embraces both cognitive and metacognitive processes. Monitoring the task and selecting appropriate strategies for the task are about the process of cognition regulation, which is another part of metacognition (Garofalo & Lester, 1985). Any problem-solving activity may be influenced by interactions among knowledge of task, person, and strategy, which also affects the regulation of cognition. Those interactions result in monitoring, revising, regulating, and evaluating possible solutions to the problem throughout the process, which leads to successful problem-solving. The NCTM (2000) stated that good problem solvers "...become aware of what they are doing and frequently monitor, or self-assess, their progress or adjust their strategies as they encounter and solve problems" (NCTM, 2000, p. 54). What makes one monitor and adjust one's problem-solving was called *reflective skills*, also known as metacognition.

A problem represents a challenging situation for some learners if they do not know how to solve it or do not have direct methods, procedures, or algorithms to solve it (Blum & Niss, 1991; Schoenfeld, 1992). Problem-solving is a goal-directed process of seeking, finding, and conducting an appropriate method to cross the gap between a learner's current state and the goal state of knowledge through problem-solving stages, as described by Polya (2004). A problem is regarded as the "difference between a goal state and a current state" (Jonassen, 2000, p. 65). To solve the problem, a learner performs complex cognitive activity by engaging multiple processes, such as cognitive, metacognitive, and self-regulatory mechanisms (García et al., 2016). The problem solver then has to use these mechanisms to find an appropriate solution to the problem.

Problem-Solving and the Cognitive–Metacognitive Framework

Problem-solving in mathematics education is based on Polya's (2004) four-stage problem-solving structure. This model and other models were developed based on Polya's model (Artzt & Armor-Thomas, 1992; Garofalo & Lester, 1985; Mayer, 1998; Schoenfeld, 1985) and they aim to explain the problem-solving process and reveal the cognitive and metacognitive structure during problem-solving. In other words, they break the problem-solving process down into pieces (Schoenfeld, 1985).

Problem-solving was investigated after Polya's (2004) development of problem-solving heuristics, which involved understanding, planning, carrying out the plan, and looking back, in order to better understand the characteristics of problem-solving (Olkin & Schoenfeld, 1994). To address this need, Schoenfeld (1985) proposed a framework and described a problem-solving process with reading analysis, exploration, planning or implementation, and verification behaviors by considering managerial or metacognitive decisions that are required for effective problem-solving (Jonassen, 2000). Although his framework emphasized the necessity of metacognitive decisions in the problem-solving process, he did not specifically explain the cognitive levels of problem-solving behaviors themselves (Schoenfeld, 1992). Garofalo and Lester (1985) also developed a framework in order to analyze metacognitive processes during problem-solving in mathematics. They described the components of these processes as orientation, organization, execution, and verification. These components resemble Schoenfeld's framework, but exploration was not included in Garofalo and Lester's framework and Garofalo and Lester (1985) did not explain in depth which specific cognitive processes needed to be analyzed in mathematical problem-solving. Therefore, Artzt and Armour-Thomas (1992) developed a framework that synthesized the cognitive and metacognitive levels of problem-solving behaviors identified by Garofalo and Lester (1985), Polya (2004), and Schoenfeld (1985), although they include similar stages or episodes (Table 1). It is also possible to combine these episodes and stages, as seen in a recent study by Jacobse and Harskamp (2012). They synthesized findings by Veenman, Kerseboom and Imthorn (2000) and Veenman, Kok and Blöte (2005) related to metacognitive behaviors while problem-solving and Schoenfeld's stages (1985). Jacobse and Harskamp (2012) did not involve cognitive behaviors in this framework.

Table 1
Problem-solving processes

Polya's stages (2004)	Schoenfeld's episodes (1985)	Garofalo and Lester's stages (1985)	Artzt and Armour-Thomas' episodes (1992)	Veenman et al. (2000, 2005)
Understanding the problem	Reading	Orientation	Read Understand	Read/ analyze/ explore
Devising a plan	Analysis Exploration	Organization	Analyze Plan	
Carrying out the plan	Planning/ implementation	Execution	Explore Implement	Plan/ implement
Looking back	Verification	Verification	Verify	Verify
–	–	–	Watch and listen	

Schoenfeld's framework was used as a starting point in this cognitive–metacognitive framework. Schoenfeld (1985) categorized problem-solving behaviors as read, analyze, explore, plan or implement, and verify. Schoenfeld's problem-solving behaviors were used to categorize the behaviors of individual students within the small group in Artzt and Armour-Thomas' framework. In this framework, the distinction between cognition and metacognition was similar to Garofalo and Lester's (1985) description that cognition was related to doing, while metacognition was related to the action of choosing the right strategies, planning, monitoring the entire process, and regulating actions.

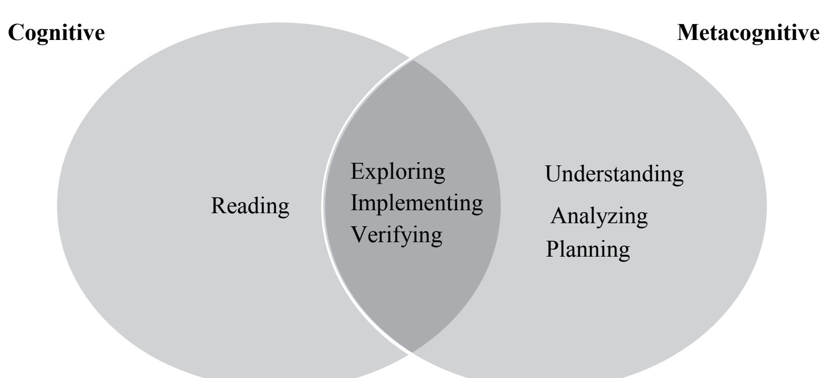

Figure 1
Episodes and levels of problem-solving behaviors (Artzt & Armour-Thomas, 1992)

It is not an easy task to differentiate cognitive and metacognitive behaviors, but the distinction based on the predominant characteristics of behaviors made by Artzt and Armour-Thomas (1992) is helpful in identifying behaviors. Metacognitive behaviors could be differentiated and observed based on the solver's statements about the process of problem-solving, whereas cognitive behaviors could be observed in the verbal or nonverbal actions of the solver. Artzt and Armour- Thomas (1992) assigned the episodes to different levels, as displayed in Figure 1.

Artzt and Armour-Thomas' framework (1992) emphasized that an action by the problem-solver can represent both cognitive and metacognitive behaviors, but one may be more predominant than the other. Based on this issue, *exploring, implementing*, and *analyzing* may represent cognitive and metacognitive behaviors according to the problem-solver's behavior. The authors emphasized that cognitive behaviors are the actual process itself. While reading, the solver can conduct other stages, such as understanding the problem by underlining the relevant components of the problem and taking notes that are predominantly metacognitive in nature. Artzt and Armour-Thomas (1992) provided an outline for the descriptions of each behavior, as seen in Table 2. This framework is still being used to analyze problem-solving behaviors (see Erbas & Okur, 2012).

Table 2
Description of cognitive and metacognitive behaviors by a problem solver

Stages	**Description of cognitive behavior**	**Description of metacognitive behavior**
Reading	Just reads the problem	–
Understanding	–	Uses domain-specific knowledge including recognition of the linguistic, semantic, and schematic attributes of the problem in his or her own words and represents the problem in a different form.
Analyzing the problem	–	Decomposes the problem into its basic elements and examines the implicit or explicit relations between the givens and the goals of the problem.
Planning	–	Selects steps for solving the problem and a strategy for combining them that might potentially lead to the problem solution if implemented.

Exploring	Executes a trial-and-error strategy in an attempt to reduce the discrepancy between the givens and the goals.	Monitors the progress of attempted actions thus far and decides whether to finish or continue working through the operation.
Implementing	Executes a strategy that grows out of his or her understanding, analysis, and/or planning decisions and judgments.	Conducts a similar metacognitive exploring process but considering time and efficiency issues based on previous experiences.
Verifying	Evaluates the outcome of the work by checking computational operations.	Evaluates the problem solution by scanning through all the stages and judging whether the outcome reflected adequate problem understanding, analysis, planning, and/or implementation.

Source: Artzt & Armour-Thomas (1992, p. 172–175)

Problem Statement and Research Questions

In line with the review by Desoete and De Crane (2019), there is a relationship between metacognition and mathematical performance and good problem solvers are expected to use metacognitive knowledge, skills, and strategies in an effective way (Jonassen, 2000; Schoenfeld, 1992). Students' mathematical performance can be improved by developing metacognition using strategies that target individual users and approaches that draw on teachers' impetus (see Baten et al., 2017; Curwen, Miller, White-Smith, & Calfee, 2010; Desoete, 2007; Özsoy, 2007; Pugalee, 2004). Preservice mathematics teacher training programs offer knowledge and skills for problem-solving and its components as an independent course or part of a pedagogical content course for teacher candidates hoping to develop their future students' problem-solving skills. Teachers who have taken part in such training can be assumed to transfer some of their problem-solving behaviors to their students. Similarly, the literature has suggested that not only mathematical achievement, but also other individual issues such as motivation might be involved in the problem-solving process (Desoete & De Crane, 2019; Jonassen, 2000; Mayer, 1998). Therefore, it may be important to study different groups of students.

Through this case study, we hope to shed light on the issue of teacher and student problem-solving behaviors in a classroom where the teacher had metacognitive knowledge related to problem-solving heuristics. Therefore, the present study aimed to investigate the problem-solving behaviors of a teacher and his students within mathematical problem-solving cases by determining the contribution of the teacher's practices to the promotion of the students' thinking skills and metacognitive functioning in problem-solving situations in accord with the systematic framework developed by Artzt and Armour-Thomas (1992). The research questions are as follows:

R1. What are the similarities and differences between the cognitive and metacognitive problem-solving behaviors of the different student groups?

- What are the similarities and differences between the cognitive and metacognitive problem-solving behaviors of the teacher-selected group and the mathematical achievement group?
- What are the similarities and differences between the cognitive and metacognitive problem-solving behaviors of the different student levels?

R2. What are the similarities and differences between the cognitive and metacognitive problem-solving behaviors of the teacher and the students?

Method

Participants

Compulsory education in the Turkish education system starts at 1st grade (age 6) and ends with 8th grade (age 14) and is divided into two levels: primary (1st–4th grade) and middle school (5th–8th grade). Pupils in each level have five hours of mathematics per week. Elementary mathematics teachers are responsible for the middle-school level. There are more public schools than private schools in Turkey. The classrooms are heterogeneous in regard to students' cognitive, affective, and motor skills.

The teacher was selected from among graduate students who had graduated from elementary mathematics education, attended a graduate course related to advance problem-solving, and worked for at least five years as an in-service teacher. He knew the problem-solving process and self-reported that he was using this problem-solving process while solving problems, which might influence the cognitive and metacognitive behavior of the students. The students were purposefully selected from the 8th-grade (13–14-year-olds). The teacher had been teaching these students for three years (six semesters), which is the maximum amount of time that a teacher spends with a given group of students at the middle-school level. Therefore, it was assumed that the students had benefited from having the highest problem-solving practices with the same teacher for three years.

Based on these criteria, we reached out to one teacher and he voluntarily participated in the study. We selected 6 of the teacher's 38 8th-grade students. These students were selected based on the criteria given in Table 3.

Table 3
Student selection criteria

	Name of the group	Criteria
Poor (P1)	MATCH (mathematical achievement group)	– Selected based on results from the national mathematics examination – Lowest performing student in the ranking (38/38)
Average (A1)	MATCH	– Selected based on results from the national mathematics examination – Average performing student (19/38)
Successful (S1)	MATCH	– Selected based on results from the national mathematics examination – Highest performing student (1/38)
Poor (P2)	TEACH (teacher-selected group)	– Selected by the teacher – Reported as an unsuccessful problem solver – Reported as having the lowest math exam scores – Reported as poorest attitude toward mathematics
Average (A2)	TEACH	– Selected by the teacher – Reported as having a moderate ability to solve mathematical problems – Reported as seeking alternate ways to solve problems but as easily distracted – Reported as having some family and friendship issues as a handicap
Successful (S2)	TEACH	– Selected by the teacher – Reported as being a problem solver in a short time – Reported as seeking alternate ways to solve problems – Reported as being eager to solve problems – Reported as being a student at the top of the rankings and having the highest scores on the midterms

In order to achieve maximum variation in the data and investigate the first research question, students were selected based on two criteria. For the first criterion, three of the six students were selected by the teacher according to his understanding of a successful, an average, and a poor student at problem-solving (the teacher-selected group; TEACH). This criterion was the result of consideration related to critics of standardized tests, who discuss whether standardized tests measure problem-solving skills (Latterell, 2003). Furthermore, some studies have considered teachers' judgments as trustworthy in assessing student achievement (Desoete, 2007). Therefore, we used the teacher's experience with the students related to their problem-solving to gather varied data. The teacher described his reasons for defining the levels and characteristics of students in the TEACH group as including that the inability to solve mathematical problems might be related to a poor level of problem-solving due to study habits, achievement level, previous educational experiences, and negative attitudes toward mathematics/problem-solving, as described in Table 3 in detail. According to the teacher, the student with a moderate ability to solve mathematical problems was talented and could find a few alternative ways of solving problems. The successful problem solver had the highest scores on both the national examination and examinations conducted by the teacher, responded first when the teacher asked a question in the classroom, and was able to find alternative methods to solve problems. The teacher believed that standardized examinations may repress students' problem-solving skills due to the overall time limitation on the exam, and he selected the students based on their skills using the problem-solving heuristics developed by Polya (2004). For the second criterion, the other three students were determined by the researchers based on their academic achievement on the national examination (the mathematical achievement group; MATCH). For the MATCH group, the successful student was at the top of the rankings, the average in the middle, and the poor at the bottom. In contrast, the selection of the TEACH group was determined by the teacher. Each group included students at successful, average, and poor levels to represent a heterogenous classroom in terms of problem-solving behaviors.

Data-Gathering Methods and Procedure

Within the process, the mathematics teacher and his six students were asked to solve one problem for practice and then three problems for analysis. The problems were selected from mathematics items in PISA tests (PISA, 2013) to evaluate problem-solving and thinking skills as PISA tests encourage the development of metacognitive skills through problem-solving (Welsh Government, 2012). Tasks in the PISA test correspond to the level of students from 7th grade to 11th grade, and the problems for this study were selected based on the required knowledge for 8th grade.

The problems were non-routine problems from different domains that assumed students could use different mathematical concepts: numbers, proportions, data. While solving the problems, they were asked to express all of the steps and strategies in their minds in order to apply a think-aloud process. Thinking aloud is one data-gathering method to understand cognitive and metacognitive processes while solving problems (Baten et al., 2017). There was no intervention while the problems were being solved except asking them to "be louder" or "keep going" to maintain the think-aloud process. Since thinking aloud requires practice, the teacher and the students were first provided a warm-up problem. Video and audio recordings were used to provide a permanent record for the coding of the problem-solving behaviors of the students and teacher. There was no time limitation for solving the tasks. Students could keep working until they were satisfied with their answers. Therefore, the tasks lasted between 5 and 30 minutes.

A semi-structured interview was used to obtain information regarding the mathematics teacher's perceptions of mathematical problem-solving processes; his previous experiences with mathematical problem-solving at university, high school, secondary school, and elementary school; the methods he uses; the types of problems he solves; and the criteria he used in selecting the students according to their mathematical problem-solving abilities and to produce some memos related to his problem-solving background. In the interview, the teacher was also asked to share his understanding of a successful problem solver and how he selected the students. This interview was required to select the teacher.

Data Analysis

Video and audio records of the students' work and teacher interviews were made. Before coding, a brief description of the students' behaviors was created (see Table 5). While analyzing the data, these transcriptions were encoded based on the cognitive and metacognitive framework of Artzt and Armour-Thomas (1992). They explicated each episode with examples in their study. These examples and definitions for each episode were considered in the analysis and encoding of the transcripts. The framework was adapted to the individual problem-solving process for this study. Watch-and-listen episodes were not evaluated because students worked on their problems individually. Table 4 breifly presents the episodes, cognitive levels, and codes. In the analysis of the task transcriptions, episodes were identified first and their codes were matched, as described briefly in Table 4. Based on the results of the matching, the cognitive level of the episodes was identified. Abbreviations for the cognitive levels were defined by the researchers with a capital letter representing the episode and a lowercase letter the cognitive level, such as R_c for reading (R) as a cognitive behavior ($_c$).

Table 4
Sample codes used for data analysis for the cognitive and metacognitive framework

Episode	Cognitive level & abbreviations	Codes
Read	Cognitive (R_c)	Reading, partial reading, rereading
Understand	Metacognitive (U_m)	Paraphrasing, highlighting
Analyze	Metacognitive (A_m)	Clarifying, visualizing,
Explore	Cognitive (E_c)	Drawing, demonstrating, selecting information
	Metacognitive (E_m)	Drawing, demonstrating or selecting information with self-monitoring, self-instruction
Plan	Metacognitive (P_m)	Making plans, selecting strategy
Implement	Cognitive (I_c)	Computing, Estimating
	Metacognitive (I_m)	Computing or estimating with self-correction, self-monitoring
Verify	Cognitive (V_c)	Checking, evaluating, confirming
	Metacognitive (V_m)	Checking, evaluating or confirming with self-evaluation, self-explaining
Watch and listen	Not assigned	(Not applicable to the current study)

The researchers worked on solving the problems, and they coded the heuristic episode and cognitive level representing the observed behavior of the participants. The warm-up problem was not evaluated. The coded behaviors were added and can be found as episodes in the Results section.

Table 5
Brief description of the students' problem-solving behaviors

	MATCH			TEACH		
	Successful (S1)	**Average (A1)**	**Poor (P1)**	**Successful (S2)**	**Average (A2)**	**Poor (P2)**
Question 1 (charts): Interpret a bar chart and estimate the number of CDs sold in the future assuming the linear trend will continue (data and uncertainty)	Read the problem aloud	Read the problem aloud	Read the problem, investigated the graph, no further step	Read the problem aloud	Read the problem aloud	Read the problem, no further step
	Found the group in the graph	Reread the problem and marked the given months (February and June)		Found the group Spinach in the table	Decided to investigate the graph first	
	Decided to determine what is asked for (average)	Found the Kicking Kangaroos on the graph (chose an incorrect bar, the light gray one)		Showed the decrease in the sales and compared other groups	Saw there was a decrease between months	
	Estimated the numbers that matched to the sales of CDs by the Kicking Kangaroos	Saw the increase month over month and estimated this increase as 200 by calculating 700 − 500 = 200		Estimated the decrease in July as approximately 500 by looking at the graph	Decided the number of the decrease was 250 and said at last 250	
	Summed the total CD sales over 6 months	Reread the problem statement (emphasizing the decrease) and said, "But it increases"				
	Divided the result by 6 to find the average	Concluded the answer was 1,100				
	Concluded the answer was 1,325					

Results

The results are presented according to the research questions, with the differences and similarities between the two student groups discussed first in terms of their cognitive and metacognitive behaviors, and the similarities and differences between the teacher and the students discussed as the next research question.

R1: The Students' Problem-Solving Behaviors

The students' problem-solving behaviors varied between cognitive and metacognitive steps. There were two distinct groups in the study. The frequency of their coded behaviors revealed some trends. The successful student in the TEACH group had 55% cognitive and 45% metacognitive behavior, while the successful student in the MATCH group had 62% cognitive and 38% metacognitive behavior. There was no difference between the metacognitive behaviors of the successful students in two groups, but they had different cognitive behaviors in terms of frequency. The frequency of metacognitive behavior for the average student in the TEACH group was 30% and for the average student in the MATCH group it was 14%. The average student in the TEACH group showed fewer metacognitive understanding behaviors, while the average student in the MATCH group showed some behaviors related to taking notes, underlining the values given in the problem. The poor students did not display any metacognitive behaviors and only read the given problem in a period of time they decided. The successful students showed most metacognitive behaviors in their groups.

All of the students started solving the problems by reading. In both groups, the students with poor problem-solving skills utilized as solving processes just reading and rereading and did not pass beyond reading the problem. Although it was beyond the scope of this study, the students were observed to have problems in reading. They tended to repeat written words and did not make any inference from the problem statements. These students did not elaborate on problems, but just read and did not interact with the information given within the problem context. Within the context of this study, the poor students did not underline the problem, did not rephrase the given problem, did not show any understanding stage of the problem. Therefore, they exhibited only reading as a cognitive behavior. On the other hand, the average and successful students in both groups were engaged with the problem, showed signs of understanding the problem, and finished the tasks with both cognitive and metacognitive behaviors (Table 6). The average problem solver in the TEACH group was relatively successful and exhibited more self-adjustment and monitoring episodes than his counterpart in the MATCH group. Moreover, the successful problem solvers in both groups demonstrated and successfully applied nearly every step in the problem-solving framework.

Table 6

The teacher and the students' problem-solving episodes

		Problem 1	**Problem 2**	**Problem 3**
MATCH	Successful	$R_c U_m R_c U_m I_c$	$R_c E_m U_m I_c$	$R_c U_m$
	Average	$R_c U_m E_c I_c$	$R_c E_c$	$R_c U_m E_m I_c$
	Poor	R_c	R_c	R_c
TEACH	Successful	$R_c U_m I_c R_c U_m P_m I_c$	$R_c U_m I_c$	$R_c U_m V_c$
	Average	$R_c U_m I_c$	$R_c E_c$	$R_c I_c$
	Poor	R_c	R_c	R_c
Teacher		$R_c U_m P_m I_c$	$R_c U_m P_m I_m$	$R_c U_m R_c U_m P_m I_m$

The students read the problems, which was a cognitive behavior. Observed reading behaviors involved looking at the diagrams and pictures in the problem. The successful students returned to reading when they got stuck solving the problem. However, the average students stayed engaged in the last episodes they were in to come up with a result. Students in both groups displayed understanding metacognitive behavior such as paraphrasing the problems; underlining significant information in the problems; taking notes on this information; using the information given; showing the information on the table, graph, or picture; and clarifying the meaning of the problem.

The students explored the problem using trial and error (cognitive) and implemented a strategy as they explored (cognitive). The students executed a strategy that grew out of their understanding and/or planned their decisions. Some of the students skipped planning and so implementation and were thus unsuccessful problem solvers since they read the problem aloud and investigated the figures and graphs, but showed no further step. They tended to conclude the solving process and write an answer without giving reasons after reading the problem aloud. Exploration (cognitive) was the behavior displayed when they did not understand the problem. "What if I did it like this?" was the characteristic differentiation between metacognitive exploration and cognitive exploration. If they found enough evidence (showing numbers or diagrams or drawing diagrams) that they understood the problem, they jumped into the implementation (cognitive) by calculating the numbers and finished the problem-solving when they found a result.

While solving the problem, the students in neither group performed metacognitive planning or verification, which is both cognitive and metacognitive. When they found a result for the problem, they, in general, did not display a verification (cognitive or metacognitive) behavior. Likewise, the students did not come up with a plan after exploration in order to solve the problem correctly and efficiently.

All in all, the **s**tudents in the TEACH (25% total metacognitive behavior) and MATCH (26% total metacognitive behavior) groups presented nearly the same proportions of cognitive and metacognitive behaviors. The successful, average, and poor problem solvers in the two groups demonstrated similar cognitive and metacognitive behaviors.

The Teacher's Problem-Solving Behaviors

Before exploring the differences and similarities between the teacher and the students' behaviors, it will be helpful to investigate the teacher's problem-solving behaviors. It was seen that the teacher tended to first read all of the problems aloud word by word. This behavior could be classified as cognitive since he just read the entire problem statement without interrupting the process. Next, he tried to show or signify the given information in the problem statement on a table, graph, or list in order to clarify the meaning of the problem by highlighting statements and recognizing domain-specific knowledge. This behavior could be stated as metacognitive understanding of the problems using the given information since he tried to explain the problem statements to himself in his own words and demonstrations. Then, he constructed plans for solving the problems using his demonstrations for the problems, which displayed understanding episodes. Therefore, this behavior was also metacognitive. Last, he implemented the plans by monitoring his progress through asking questions such as "…where does this attempt take me?". The teacher's implementation episodes involved both cognitive and metacognitive behaviors. He implemented his plans systematically while monitoring his solving processes and regulating the plan when needed for two problems and implemented his plans without monitoring his work and solution process for one problem. Table 6 presents sample problem-solving behaviors by the teacher for one problem.

Table 6
An example of the teacher's behaviors for understanding, planning, and implementing.

Episode	Coding	Behavior
Understanding the problem (metacognitive)	U_m	Soo, The distance between M and there… (*shows the distance by drawing an arrow*). If the river bed is there…. If M is there… It (the problem) asks how many meters it is (*shows the area in the picture that the problem is asking about*).
Planning the problem (metacognitive)	P_m	Now, to find the result, it was given as 10 meters. We need to find the distance between M and P.
Implementing (metacognitive)	I_m	What is there between M and P? Now, since point M is the center of the Ferris wheel ,what does that tell us? (*shows the radius of the Ferris wheel in the problem*) What is the radius? We already know the radius is half of the diameter. Ahh... the problem gave us the diameter and we need to find radius. 140 divided by 2, we get 70 meters (*calculates on paper*). Now, if r is 70 meters, there is 10 meters in here (*shows the distance between point M and the bottom of the river bed*). 70 plus 10, the result is 80 meters.

In the interview, he defined his problem-solving process which he applies in class as follows:

> I read the problem to students as quickly as possible since our time is generally limited, and I ask what we have and what we need to find. Then, I ask what we can do, and it actually forms our plan. After that, we discuss some plans and we employ one of the plans and check whether our result is true. In this process, I try to allow the students to explore the situation with individual-work or group-work.

Generally, his problem-solving process is similar to Polya's (2004) problem-solving steps except for one difference. He stated that he always tries to find some different situations or a different solving process while solving problems. Therefore, according to his solving process, he applied an extension phase at the end of the process.

R2: The Teacher's and the Students' Problem-Solving Behaviors

Each student showed different episodes for each problem. Although there may have been behaviors that were coded the same, they represented individual-dependent distinct behaviors. Analysis of the transcriptions indicated that students with different problem-solving skill levels exhibited different frequencies of cognitive and metacognitive behaviors with the PISA problems. Students who were selected as successful problem-solvers had more frequent metacognitive behaviors compared to their groups. The frequency and episodes of metacognitive behaviors were found to be similar for the same level in two groups. The number of metacognitive behaviors decreased in terms of the levels within the groups. The most common cognitive behavior, which was observed in both the teacher and the students, was reading the problem. The teacher also tended to reread the problem as the successful students did to go on to a further step. The most common metacognitive behaviors, which were observed in both the teacher and the successful problem-solvers, involved understanding the problem (clarifying the meaning of the problem, recognizing domain-specific knowledge, identifying data on the graph).

While solving the problems, the teacher explored, consistently planned, and implemented metacognitively. That is, while exploring he explained his actions and monitored his progress; he planned what to do next to reach a result. Such consistency across each task was not observed in the students' episodes. Verifying and planning episodes were not observed in the students in either group. Although planning was seen in the teacher's think-aloud process, it was not traced in the students' episodes, which means the students' problem-solving did not reflect their teacher's solution.

Discussion and Implications

The results indicated that the students and the teacher followed a path involving reading, understanding, exploring, planning, and implementing, which was consistent with the literature (Artzt and Armour-Thomas, 1992; Erbas & Okur, 2012; Kuzle, 2013). Furthermore, not all episodes occurred in each problem-solving task and the behaviors that represented the episodes changed according to the participants. It is not new to say that problem-solving episodes are task- and individual-dependent (Erbas & Okur, 2012; Kuzle, 2013).

Just as Schoenfeld (1981) claimed that, in contrast to average problem-solvers, whose sequence of heuristics was only reading and exploring, expert problem solvers return several times to different heuristic steps, the average participants in this study also displayed this kind of behavior. Each participant started to solve the problem by reading as a habit, but rereading emerged in this study based on a specific focus on solving the problem such as to clarify what was given or to see the problem statement, which might mean self-evaluating understanding. Although Artzt and Armour-Thomas (1992) considered reading as a cognitive process, there should be a differentiation between reading and rereading in terms of cognitive levels.

Verification is another concern to be discussed for this study. As we reported, neither the teacher nor the students displayed verification, in other words, looking back (Polya, 2004). Verification is an episode for not only checking the answer but also improving the problem-solving experience, which means encouragement to find other solution strategies (Polya, 2004) Verification being absent from the episodes is not a new result for this study (Erbas & Okur, 2012; Kuzle, 2013). Cai and Brook (2006) explained this issue as learners' hastiness to finish the problem by reaching the result. This might be the case for the students, but the teacher being a participant knowledgeable about the problem-solving process might suggest other issues relating to teacher education. According to Curwen et al. (2010), teachers' metacognition about their practice leads upper elementary grade students to higher learning goals by developing the students' metacognition and reflection about their thinking, exploration of ways of solving and deep understanding in content domains, and integration of literacy in content areas. Such training increased the performance of students in mathematical problem-solving and this training had a sustained effect on mathematical problem-solving. However, training of metacognitive skills must be done explicitly by teachers (Desoete, 2007).

Previous studies have already revealed that there is a relationship between mathematical performance and metacognitive behaviors (Özsoy, 2011; Schneider & Artelt, 2010). Here, the successful problem solvers showed more metacognitive behaviors than other participants in their groups. Their selection criteria were a national examination and teacher observation. Based on the episodes in the behaviors of the students at the same levels in the two groups, it can be concluded that they were similar. National examinations and achievement in mathematics have been used as a selection criterion for participants in metacognition studies (Erbas & Okur, 2012). It might be worth further discussion to consult a teacher's observations about the metacognitive behaviors of his/her students.

There were also differences in the metacognitive behaviors that the teacher displayed but the students did not, such as planning. The role of the teacher is to be emphasized in terms of the metacognitive aspect of problem-solving. Some experimental studies have found significant changes in students' problem-solving tasks in terms of metacognitive behaviors after according instruction from teachers (Curwen, et al., 2010; Desoete, 2007). For example, Desoete (2007) found in her study that metacognitive skills were trainable and students were able to learn to adapt a more orienting and self-judging learning approach after brief metacognition training. However, studies have not revealed more detail about the development of poor students' problem-solving behaviors. They need help to go beyond reading the problems. The role of the teacher for different levels of problem solvers could be identified and further studies should be conducted.

Limitations and Recommendations

This study was limited to seven participants with six students and a mathematics teacher from the 8th grade. Therefore, it constitutes a pilot study for further studies that will consider teacher–student interactions in terms of cognitive and metacognitive behaviors. Similar studies may be conducted with other education levels and their results may be compared and contrasted to strengthen or refute the argumentation provided in this study. Another limitation was that this study relied on self-reporting by the teacher and a non-routine problem-solving process by the students. There was no classroom observation and the selection of the TEACH group relied on the teacher's definition of successful, average, and poor students.

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