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STUDIA MINORA FACULTATIS PHILOSOPHICAE UNIVERSITATIS BRUNENSIS
K 1 (1979) – BRNO STUDIES IN ENGLISH 13

THE ORDERED-TRIPLE THEORY CONTINUED

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INTRODUCTION

In our previous paper entitled An ordered-triple theory of language (Brno Studies in English 12), we have voiced some more or less general views on how to both generate and analyze expressions of a natural language if we take into account all the three components of semiotics: syntactics, semantics and pragmatics. Since 1973, when the manuscript was submitted to the editor, our team have focussed efforts on (i) elaborating the system of semantics and semantic analysis and (ii) testing a computer program of syntactic analysis compatible with the designed systems of semantic and pragmatic analyses.

Since we regard semantics as the keystone of our theory, we have paid much attention to the choice and development of a system of logical semantics satisfying the requirements of the semantics of a natural language. It is our team's logician, P. Materna, who assumed the responsibility for choosing Tichý's intensional logic and shaping it according to our demands. Most relevant sections of this semantic theory form the core of Part One of the present paper.

The computer program of syntactic analysis has been designed and tested by K. Pala, who also supervised the work of our programmers I. Palová and P. Čihánek. The results of their endeavour can be found in Part Two.

PART ONE

Chapter I

AGAINST EXTENSIONALISM

To solve the problem of an adequate analysis of language expressions is hardly thinkable without a carefully elaborated theoretical conception, which – in our opinion – is also a conditio sine qua non for the subsequent computer simulation of some important fragments of language communication. According to Morris (1946), any general theory of language has to make the distinction between syntactics, semantics, and pragmatics. In addition to that, such a theory has also to study the interplay of the above three components. As has been said before, Part One is mainly to deal with the semantic component of a general theory of linguistic analysis, its main task being to find and elaborate a method that would enable us to decipher what the language expressions are about; possessing this method would result in our ability to discover for any particular language the laws connecting the surface structures of its expressions, particularly words, phrases, and clauses, with their "meanings", i.e., with what these expressions speak about.

Whereas the work of discovering such laws concerning particular languages would be a job for the linguist, the general theory itself is some sort of methodological framework. One would expect, therefore, that working out a great part of this framework should be in the competence of logic. Indeed, the study of syntax of the "artificial languages" of logic necessarily differs from the study of syntax of natural languages, but what is standardly named "semantics" (in the Morrisian sense) should be independent of the specific features of particular languages. Yet every specialist in this area is aware of the traditional tension between logical semantics and "linguistic semantics". We do not intend to analyze the reasons of this tension here, but some aspects of it are probably important.

One of the most frequent objections raised by the linguists to the competence of logic in supporting linguistics with a general methodological framework consists in accusing the logicians of a systematic simplification of the real problems of linguistic analysis. On the other hand, it is possible to reproduce some complaints made by logicians, concerning the inability of the linguists to pass from empirical description to a deep analysis of the meanings of language expressions. Here we should like to point out that for the greater part this situation stems from some gross errors in the approach of standard logic to the semantic analysis of natural languages. Briefly, the source of these errors can be called "extensionalism". Extensionalism may acquire the form of a manifest confession (this is the case of, say, Quine) or it is a hidden feature of a logical theory (in this sense even Frege, Church, the modal logicians, Montague, etc., can be branded with it).

We have drawn our main theoretical inspiration from the conception contained in some papers and, primarily, in a hitherto unpublished manuscript by Pavel Tichý (1976). In Part One of the present paper all the basic ideas concerning what is traditionally called "semantics" are those offered by Tichý

(1976). In Chapters I–IV we shall not therefore use such formulations as "according to Tichý" or "after Tichý". In most cases, we consider phrases like "in our opinion" or "to our mind" superfluous because we intend to avoid any statements that would only depend on our subjective intuitions and were not founded on a persuasive argument accessible to objective control.

A moderate form of extensionalism is what might be called "Fregean-Churchian semantics" (cf. Frege 1892, Church 1956). According to Fregean-Churchian semantics, language expressions denote some objects (extensions, i.e., individuals, classes, relations, truth-values) on the one hand and express some intensions (Frege: "Sinn"; Church: "sense", "intension", "concept") on the other. Thus individual constants denote individuals and express individual concepts; common nouns denote classes or relations-in-extension and express properties or relations-in-intension; sentences denote truth-values and express propositions.

Taking, e. g., the sentence

(1) Prague is the capital of Czechoslovakia

we can say that according to Fregean-Churchian semantics, (1) denotes truth and expresses the proposition that Prague is the capital of Czechoslovakia; "Prague" denotes Prague (an individual) and expresses perhaps nothing else because "Prague" is a proper name; "the capital of Czechoslovakia.

Without taking intensions into account (i.e., remaining within the sphere of "pure extensionalism"), we are not able to analyze (1) so as to save the obviously empirical character of it. Indeed, if "Prague" as well as "the capital of Czechoslovakia" denotes Prague and if "is" means identity (which is obvious, too), then (1) claims the identity of an object with itself, which is no empirical claim, of course. The analysis of an analogous example led Frege to the idea of establishing the realm of intensions in addition to the realm of extensions, but, unfortunately, his approach is not a solution to the problem either. (By the way, this problem is a general one concerning the semantic analysis of any empirical sentence.) Frege's notion of intension being expressed (in contrast to extension being denoted) is in so far indetermined as it cannot play any essential role in the semantic analysis of language expressions. Another weak point of extensionalism consists in that one of its consequences is fully unacceptable if we want (which is certainly the case) to distinguish between understanding a sentence and knowing its truth-value. It is perfectly clear that a vast majority of sentences we are able to understand is such that we do not know their truth-value. Yet, if we accept a quite intuitive principle according to which we cannot understand any expression unless we understand every component of it and if "to understand the expression A" means "to know what the expression A is about" (which also appears to be an intuitive principle), we come to the conclusion that for an extensionalist it is impossible to understand any sentence without knowing its truth-value (because a sentence is, according to him, about a truth value) and that a consistent extensionalist could not understand, e.g., the sentence

(2) The rector of Charles' University is a smoker

without knowing who the rector of Charles' University actually is and without knowing which are the members of the "class of smokers".

Thus it is necessary to refuse any form of extensionalism and to establish the connection between expressions and intensions as the fundamental semantic relation performing the role of the denoting (or: naming) relation. In consequence, (2) names not a truth-value but a proposition, "the rector of Charles' University" names not a definite person but the rector of Charles' University (which is an intension, too), "smoker" names not a class but a property. The proposition named by (2) informs us about a connection between the office of the rector of Charles' University and the property of being a smoker. The actual person occupying the office comes into play, not during the process of understanding (2), but during the process of verifying (2). Similarly, "the capital of Czechoslovakia" names an individual concept (analogous to the concept of the rector of Charles' University). (1) as well as (2) ceases to get a trivial and counterintuitive semantic interpretation. Take, e. g., (2). While someone's being or not being an element of a class is wholly dependent of experience (a class is simply determined by the elements it contains), his having or not having an empirical property (such as being a smoker) is empirically testable.

Remark: The word "Prague" is a specific case. Like other proper names, it directly names an individual, i. e., something that is an extension, Nevertheless, such cases are incorporable into the general intensionalist conception if we define extensions as "intensions of zero order".

Chapter II

INTENSIONS

The most important feature of the present semantic theory is that it is consistently intensionalist. Before proceeding any further, we have to explain in more detail what is meant by "intensions". What follows is an ontology that will enable us to construct exact definitions of the basic concepts of the theory.

The objects a language L is able to speak about are constructible over an "epistemic basis" (EB) with respect to L. To define such a basis, we informally introduce an auxiliary concept: the concept of "intensional basis" (IB) with respect to L. We suppose that the users of L have at their disposal an IB, i.e., a common collection of elementary mutually independent empirical tests and other means enabling them to decide whether the objects (and the strings of objects) established over what is called the universe of discourse (U) do or do not exhibit some empirical traits. Thus nothing is known a priori, i.e. before applying the members of IB to the objects over U, about the distribution of the empirical traits through U. Some distributions are, of course, impossible: it is impossible for an element of U simultaneously to exhibit and not to exhibit some trait. All other distributions are possible (conceivable) and we call the set of them "the logical space" or "the set of possible worlds" (relative to L). One of the possible worlds is the actual one. We would learn which of them it is if we applied all the members of IB to the (strings of) objects over U.

Now we can introduce the concept of minimal epistemic basis (MEB) relative to L. It is a collection of three mutually disjoint non-empty sets:

(i) the set ι (iota) is the universe of discourse relative to L and its members are individuals;

(ii) the set o (omicron) is the set of truth-values and its members are T (truth) and E (falsity);

(iii) the set μ (mu; in Tichý's manuscript ω, omega) its the set of possible worlds relative to L, and its members are all the possible distributions of empirical traits among the objects over ι, i.e. all the possible series of members of o as answers to applying the members of IB to the objects over U. An EB is any such collection of mutually disjoint non-empty sets as contains an MEB as its subcollection.

Let B be an EB. We define a type over B as follows:

(i) Every member of B is a type over B. (Thus the types over an MEB are ι, o, μ.)

(ii) Where α, $\beta_1, \ldots, \beta_n$ are types over B, $\alpha(\beta_1, \ldots, \beta_n)$ is a type over B; we conceive of $\alpha(\beta_1, \ldots, \beta_n)$ as the collection of all the functions that associate every member of $\beta_1 \times \ldots \times \beta_n$ with at most one member of α. Thus – if B is an MEB – $o(\iota)$, $o(\iota)(\mu)$, $o(\iota,o(\mu))(\mu)$, $o(o(\iota))$, etc., are types over B; $o(\iota)$ is the collection of all the functions from ι into o (including partial functions), $o(\iota)(\mu)$ is the collection of all the functions from μ into $o(\iota)$, $o(\iota,o(\mu))(\mu)$ is the collection of all the functions from μ into the set of all the functions from $\iota \times o(\mu)$ into o, $o(o(\iota))$ is the set of all the functions from $o(\iota)$ into o, etc.

(iii) There are no types over B except those definable by (i) and (ii).

It follows from the definition of types over B that the hierarchy of these types is infinite for any B.

Wherever we suppose an EB to be fixed, we can omit "over B" when speaking of types.

Let α be a type. Any member of α will be called an α-object.

Thus $o(\iota)$-objects are classes of individuals because there is no difference between a class of members of ι and the characteristic function of this class. (Usually, we take into account only those members of $o(\iota)$ that are total functions.) Similarly, $o(\iota, \iota)$-objects are binary relations-in-extension of individuals, $o(\iota,o(\iota))$-objects are binary relations-in-extension between individuals and classes of individuals, etc.

This modification of Church's simple theory of types (Church 1940), together with the above intuitions concerning IB and with the concept of MEB, enables us to give an exact definition of intensions, so that it may satisfy the current instuitions connected with the term "intension":

(i) Let α be a type such that there is no β such that $\alpha = \beta(\mu)$. In this case α-objects are intensions of 0-th order or extensions.

(ii) Let α-objects be intensions of k-th order. Then $\alpha(\mu)$-objects are intensions of $(k + 1)$-th order.

Thus whereas $o(\iota)$-objects are extensions (classes of individuals), $o(\iota)(\mu)$-objects are intensions of the 1-st order. Or: the truth-values are o-objects, i. e., extensions, whereas $o(\mu)$-objects are intensions of the 1-st order and $o(\mu)(\mu)$-objects are intensions of the 2-nd order.

Later on (in Ch. IV.) we shall be in need of a concise notation for what will be termed linguistic constructions. For this reason we shall define here the way of writing some kinds of type:

Let α be a type. (i) $\alpha^0 = \alpha$;
(ii) $\alpha^{k+1} = \alpha^k(\mu)$.

Examples: $o(\iota)^0 = o(\iota)$, $o(\iota)^1 = o(\iota)(\mu) = o(\iota)(\mu)^0$, $o(\iota)\mu)((\mu) = o(\iota)(\mu)^1 = = o(\iota)^2$.

Notice that the intensions of k-th order where $k > 0$ are functions from μ. This is exactly what is desirable from the viewpoint of our intuitions, for we wish to conceive of intensions (in accordance with, e. g., Frege) as something that makes it possible to identify extensions. Take, e. g., properties of individuals as an instance of intensions. What is a property, say, of being black, of being a smoker, etc.? Obviously, we know such a property not because we know all the objects which are black, which are smokers, etc.; but we can decide for any state of affairs (or: in any possible world!) whether a given object is black, whether it is a smoker, etc. (The ability to make such a decision may only be present theoretically, in principle, and need not be technically realizable.) In other words, if we were given some possible world, we could generate a class of individuals that satisfy the criterion given by our knowledge of the property in question. Thus a property of individuals can be identified with a function that associates any possible world with a class of individuals. In a similar way it can be shown that a proposition is something that enables us to identify the truth-value of a certain condition, and consequently, propositions are $o(\mu)$-objects. The capital of Czechoslovakia is another example of an intension: it is an individual concept and therefore an $\iota(\mu)$-object: this function associates some possible worlds (including the actual one) with Prague, some other possible worlds with, say, Brno, etc.

Have a table of some important intensions of the 1-st order. We suppose that $\alpha, \beta_1, \ldots, \beta_n$ are any types.

Type	Objects	Corresponding extensions, i. e. extensions that are values of the intensions in a given possible world
$\iota(\mu)$	individual concepts	individuals
$o(\mu)$	propositions	truth-values
$o(\iota)(\mu)$	properties of individuals	classes of individuals
$o(\underbrace{\iota, \ldots, \iota}_{n})(\mu)$	n-ary relations-in-intension between individuals	n-ary relations-in-extension between individuals
$o(\alpha)(\mu)$	properties of α-objects	classes of α-objects
$o(\beta_1, \ldots, \beta_n)(\mu)$	relations-in-intension between $\beta_1, \ldots, \beta_n$-objects	relations-in-extension between $\beta_1, \ldots, \beta_n$-objects

As an example of an intension the type of which is presented in the last "type" row of the above table, we can adduce believing: it is obviously an $o(\iota, o(\mu))(\mu)$-object, i. e. a relation-in-intension between individuals and propositions. When applying this function to the actual world, we get the members of the corresponding relation-in-extension, i. e. the ordered pairs the first members of which are individuals and the second members of which are the propositions such that the first member believes the second member to be true.

Defined in this way, the intensions cease to be "obscure entities" (Quine) and become well-defined objects (provided functions are considered to be well-defined objects).'

A language is determined by EB, vocabulary and grammar. This means that once a language is given, no question concerning membership in one of the members of EB can arise. For example, for every user of a language, the individuals are given a priori. Furthermore, the universe of discourse is the same for all the possible worlds, so that we do not suppose there is any possible world, say W, with respect to which the universe of discourse contains a certain individual or individuals that with respect to another possible world, say W', are not contained in the universe of discourse. Thus if Pegasus exists in some possible world W and does not exist, e.g., in the actual world, it only means that – according to the present conception – some individual, say x, does in W, and does not in the actual world, match the characteristic of Pegasus. (It should be clear by now that Pegasus is an $\iota(\mu)$-object and not an ι-object.)

Chapter III

CONSTRUCTIONS

The vocabulary and grammar of a language L are – *qua* components of L – connected with EB. Now another important feature of the present theory consists in that this connection is conceived of not as a direct connection but as a connection mediated by what is to be called constructions. In the following paragraphs we intend to explain what is meant by constructions in our theory.

We shall begin with some examples. Let our MEB be extended so as to contain the set ν of natural numbers (0, 1, 2, . . .). Whereas 3, 5 are ν-objects, + (plus), : (divided by) are $\nu(\nu,\nu)$-objects: they are functions associating any pair of natural numbers with at most one natural number; + associates, e. g., the pair $\langle 3,5\rangle$ with 8, : associates $\langle 8,2\rangle$ with 4, etc. (Notice that : associates, e. g., $\langle 8,3\rangle$ with no natural number and that the same can be said about any pair the second member of which is 0.) Knowing that 0, 1, 2, 3, . . . are ν-objects and +, :, . . . are $\nu(\nu, \nu)$-objects, we put the following question:

What sort of entity is $3 + 5$?

Clearly, it is neither a ν-object nor a $\nu(\nu,\nu)$-object. We claim, therefore, that such entities as $3 + 5$, $7 : 2$, $7 : (3 + 4)$, etc., are no objects at all. They serve to construct objects if it is possible. Hence we call them constructions. A construction always constructs at most one object. Generally speaking, this object is different from the construction itself: $3 + 5$ is something different from 8, because the former is compound and contains occurences of 3, 5, and $+$, whereas 8 is a simple object. Nevertheless, we can conceive of objects as an "extreme" case of constructions, namely as self-constructing constructions. (It is important to remember that when speaking about constructions we do not consider them to be inscriptions. Thus saying that $3 + 5$ is a construction, we do not mean by this that the inscription "$3 + 5$" is a construction. The constructions themselves are independent of any language.)

Before we define exactly the notion of construction, we shall generalize our intuitions. For the sake of this generalization, the following question is of some interest to us:

Is $3 + x$ a construction?

By x we mean an abstract representative of ν-objects; in other words, any ν-object can be thought of as standing in place of it. Let such abstract representatives of (α-)objects be called (α-)variable (where α is a type). Now, $3 + x$ constructs no object, as it stands, but with, say, 5 in place of x, it constructs 8, with 7 it constructs 10, etc. Such entities that differ from constructions (as characterized up to now) only in that they contain at least one occurrence of a variable (which means that what they construct depends on what object is in place of such a variable), will be called open constructions. We can suppose that there are denumerably infinite sets of α-variables for any type α at our disposal. Any (total) function that associates every α-variable with exactly one α-object will be called valuation. The value of a valuation v at the variable a will be called the v-instance of a. The v-instance of a may be said to be v-constructed by a. Let A be the object that is constructed by an open construction A if all the occurrences of those variables on which it depends what object (if any) will be constructed by A are replaced by v-instances of them. Then we say that A v-constructs A. If A does not construct any object after such a replacement, we say that A is v-improper.

Examples: Let A be $3 + (8:x)$. Let v_1 replace x by 4. Then A v_1-constructs 5. Let v_2 replace x by 0. Then A is v_2-improper.

Let A′ be $x - x$. Then A′ v-constructs 0 for every valuation v.

Let A″ be $\frac{x + y}{5 - x}$. A″ is v-improper, e. g., for any valuation v that replaces x by 5. Let v_1 replace x by 4 and y by 3. Then A″ v_1-constructs 7.

The above basic notions need not be exemplified by such arithmetical examples only, of course. Later on, we shall return to MEB or to some extensions of MEB. Now we can define the general notion of construction.

A terminological note: Variables and objects will be called atoms here. The notions of type and construction have to be relativized to an EB. The explicit relativization will be omitted in the present definition.

Let $\alpha, \beta_1, \ldots, \beta_n$ be types.

(i) Every α-atom is an α-construction. If the atom b is an α-object B, then b v-constructs B for any valuation v. If the atom b is an α-variable b, then b v-constructs the v-instance of b.

(ii) Let $F_0, F_1, \ldots, F_n$, $n \geq 1$, be $\alpha(\beta_1$-$, \ldots, \beta_n)$-, β_1-$, \ldots, \beta_n$-constructions, respectively. Then $F_0(F_1, \ldots, F_n)$ is an α-construction called the application of F_0 to $\mathrm{F}_1, \ldots, \mathrm{F}_n$. If at least one of $\mathrm{F}_0, \mathrm{F}_1, \ldots, \mathrm{F}_n$ is v-improper, then $\mathrm{F}_0(\mathrm{F}_1, \ldots, \mathrm{F}_n)$ is v-improper. Otherwise let $F_0, F_1, \ldots, F_n$ be the objects v-constructed by $\mathrm{F}_0, \mathrm{F}_1, \ldots, \mathrm{F}_n$, respectively. Then $\mathrm{F}_0(\mathrm{F}_1, \ldots, \mathrm{F}_n)$ is v-improper if F_0 is not defined at the n-tuple $\langle F_1, \ldots, F_n\rangle$; otherwise $\mathrm{F}^0(\mathrm{F}_1, \ldots, \mathrm{F}_n)$ v-constructs the value of F_0 on the n-tuple $\langle F_1, \ldots, F_n\rangle$.

(iii) Let $x_1, \ldots, x_n$, $n \geqq 1$, be β_1-, $\ldots$, β_n-variables, respectively. Let Y be an α-construction. Finally, let $\mathrm{v}(A_1, \ldots, A_n/x_1, \ldots, x_n)$ be a valuation that differs from the valuation v at most in that it replaces $x_1, \ldots, x_n$ by the β_1-, $\ldots$, β_n-objects $A_1, \ldots, A_n$, respectively. Then $\lambda x_1 \ldots x_n$ (Y) is an $\alpha(\beta_1, \ldots, \beta_n)$-construction called the $x_1, \ldots, x_n$-abstraction of Y. This kind of construction v-constructs the function F defined as follows: Let $A_1, \ldots, A_n$ be β_1-, $\ldots$, β_n-objects, respectively. If Y is $\mathrm{v}(A_1, \ldots, A_n/x_1, \ldots, x_n)$-improper, then F is not defined at $\langle A_1, \ldots, A_n\rangle$. Otherwise the value of F is the object that is $\mathrm{v}(A_1, \ldots, A_n/x_1, \ldots, x_n)$-constructed by Y.

(iv) Nothing else is an α-construction.

Examples: Let MEB contain $\mathrm{U} = \{A, B\}$.

We define the following functions:

G_1 is an $o(\iota,\iota)$-object defined as flollows:

	G_1	
A,A	T	(truth)
A,B	F	(falsity)
B,A	–	(not defined)
B,B	T	

G_2 is an $\iota(\iota,\iota)$-object defined as follows:

	G_2
A,A	A
A,B	A
B,A	B
B,B	–

Let x,y be ι-variables and let x v_1- and v_2-construct A and v_3- and v_4-construct B, y v_1- and v_3-construct A and v_2- and v_4-construct B.

Then (a) $G_2(x,A)$ is an application of G_2 to x,A. Similarly, (b) $G_2(x,y)$ is an application of G_2 to y, x. We can see that (a) v_1- and v_2-constructs A, v_3- and v_4-constructs B. (b) v_1- and v_3-constructs A, v_2-constructs B, and is v_4-improper.

Let us have the following construction:

(c) $G_1(G_2(x,A), G_2(y,x))$.

Since G_1 is an $o(\iota,\iota)$-atom, and therefore an $o(\iota,\iota)$-construction, and $G_2(x,A)$, as well as $G_2(y, x)$, are ι-constructions, (c) is – qua an application of G_1 to (a) and (b) – an o-construction. Clearly, (c) v_1-constructs T, v_2-constructs F and is v_3- and v_4-improper.

Let us have the following construction:

(d) $\lambda x(G_1(G_2(x,A), G_2(y,x)))$.

(d) is an x-abstraction of (c) and is therefore an $o(\iota)$-construction. Since (c) $\mathrm{v}_1(A/x)$-constructs T and is $\mathrm{v}_1(B/x)$-improper, (d) v_1-constructs the following function, say, H_1:

	H_1
A	T
B	—

Similarly, it can be shown that (d) v_2-constructs the function, say, H_2:

	H_2
A	F
B	—

etc. Finally, observe

(e) $\lambda xy(G_1(G_2(x,A),\ G_2(y,x)))$.

(e) is an $o(\iota,\iota)$-construction (it is the x,y-abstraction of (c)). We can see that for any v, (e) v-constructs the following function, say, H_3:

	H_3
A,A	T
A,B	F
B,A	—
B,B	—

From our definitions it follows that
(1) what an $x_1, \ldots, x_n$-abstraction v-constructs never depends on the variables $x_1, \ldots, x_n$;
(2) $x_1, \ldots, x_n$-abstractions are never v-improper.

Ad (1): Thus (e) is not an open (but a closed) construction.

Having defined the notion of construction, we can say the following:

Let L be a language. The expressions of L express some kinds of construction (over an EB) and denote what is constructed by these constructions. (In case the construction in question is an atom, what is denoted is identical with what is expressed.) Thus we can say that expressions of L express α-constructions where α-objects are intensions of the k-th order, $k \geq 0$.

To analyze semantically (or: to offer a semantic analysis of) an expression A (of L) means to find the construction expressed by A.

Chapter IV

LINGUISTIC CONSTRUCTIONS

The general features of our semantic theory have been set forth in the preceding two chapters. Any non-indexical (for this term see Chapter VI) expression of a language L (except for the "syncategorematic" expressions) names (denotes) an intension (of the k-th order, $k \geq 0$) and expresses a construction that constructs the intension in question. Now, it is probable that not every thinkable construction over an EB is expressible by the expressions of a language L. We shall define the class of what (after Tichý) we should like to call "linguistic constructions". The expressions of any language L will express just the members of this class.

Before defining linguistic constructions, let us introduce a sort of shorthand for the notation of some constructions:

Let w be a μ-variable. If A is an α^m-construction where α is a type of an extension, we write

(i) $\mathrm{A}(w)^0 = \mathrm{A}$,

(ii) $\mathrm{A}(w)^{k+1} = \mathrm{A}(w)^k(w)$,

where $k < m$.

Example: Let K be an $o(\iota)(\mu)(\mu)$-object, i. e., an $o(\iota)^2$-object. Then $(Kw)^0 = K$, $K(w)^1 = K(w)$, $K(w)^2 = K(w)(w)$.
$K(w)^1$ is an $o(\iota)(\mu)$-construction, $K(w)^2$ is an $o(\iota)$-construction.

Definition of linguistic constructions:

(i) Let w be the alphabetically first μ-variable. Every atom different from w is a linguistic construction.

(ii) Let $\mathrm{F}_0, \mathrm{F}_1, \ldots, \mathrm{F}_n$, $n \geq 1$, be $\alpha(\beta_1, \ldots, \beta_n)^{i_0}$-, β^{i_1}-, $\ldots$, β^{i_n}-linguistic constructions, respectively, where i_j for $j = 0, 1, \ldots, n$, is 0 or 1; if $i_r = 1$, then let F_j not be a variable. Then

$\lambda w(\mathrm{F}_0(w)^{i_0}(\mathrm{F}_1(w)^{i_1}, \ldots, \mathrm{F}_n(w)^{i_n}))$

is a linguistic construction called $i_0, \ldots, i_n$-compound of $\mathrm{F}_0, \mathrm{F}_1, \ldots, \mathrm{F}_n$.

(iii) Let $x_1, \ldots, x_m$, $m \geq 1$, be distinct variables different from w. Then if Y is a linguistic construction,

$\lambda x_1 \ldots x_m(\mathrm{Y})$

is a linguistic construction called a O-abstract of Y.

(iv) Let $x_1, \ldots, x_m$ be as in (iii) and let Y be an α^1-linguistic construction. Then

$\lambda w(\lambda x_1 \ldots x_m(\mathrm{Y}(w)))$

is a linguistic construction called a 1-abstract of Y.

(v) The set of linguistic constructions is the minimal set containing constructions satisfying (i) – (iv).

Examples: Let O be a relation-in-intension of being older (than), i.e., an A, B be two members of U, i.e., individuals (ι-objects). Let C be the individual concept of the (present) capital of Czechoslovakia (i.e., an $\iota(\mu)$-object). Then

(a) $\lambda w((wO)(A,B))$ $(= \lambda w(O(w)^1(A(w)^0, B(w)^0)))$ is a $1-0-0$-compound of O, A, B. We can see that (a) is an $o(\mu)$-construction constructing the proposition that A is older than B.

(b) $\lambda w(O(w)(C(w),B))$

is a $1-1-0$-compound of O, C, B. Again, (b) constructs a proposition: the capital of Czechoslovakia is older than B.

(c) $\lambda x(\lambda w(O(w)(x,A)))$,

where x is an ι-variable, is a 0-abstract of

(c') $\lambda w(O(w)(x,A))$,

because (c'), being a $1-0-0-$compound of O, x, A, is a linguistic construction.

(c) is an $o(\mu)(\iota)$-construction (cf. the definition of $x_1, \ldots, x_n$-abstraction in Chapter III). It constructs a function that associates every individual with the proposition that this individual is older than A.

(d) $\lambda w(\lambda x(\lambda w(O(w)(x,A))(w)))$

is a 1-abstract of

$\lambda w(O(w)(\mathrm{x},A))$ $(= \mathrm{c}'))$;
clearly, it is an $o(\iota)(\mu)$-construction constructing the property to be older than A.

(e) $\lambda w(\lambda xy(\lambda w(O(w)(x,y)))(w))$
is a 1-abstract of
$\lambda w(O(w)(x,y))$.
(e) constructs the relation-in-intension of being older (than). We can see that (e) is equivalent to O (in that it constructs the same object as O).

Reducts:

It can be proved that the construction
$\lambda w(\mathrm{Z})(w)$,
where Z is a construction, is equivalent to Z. Therefore, we can — where possible and necessary — replace the linguistic constructions by their reducts. The reduct of a 1-abstract
$\lambda w(\lambda x_1 \ldots x_n(\mathrm{Y}(w)))$,
where Y is of the form $\lambda w(\mathrm{Z})$, is the result of replacing $\mathrm{Y}(w)$ by Z. The reduct of a linguistic construction A is the result of performing all such replacements within A.

Thus the reduct of (d) will be
(d′) $\lambda w(\lambda \mathrm{x}(O(w)(\mathrm{x},\mathrm{A})))$
and the reduct of (e) will be
(e′) $\lambda w(\lambda xy(O(w)(x,y)))$.

As has been said before, a language L is determined by EB, vocabulary (lexicon) and grammar. Grammar may be conceived of as a set of rules that connect expressions of L with the constructions expressed by them, i.e., with their analyses. The vocabulary contains both words (phrases) expressing (= denoting) atoms, indexical words (see Chapter VI) and syncategorematic words, the role of which is purely syntactic (they neither express nor denote anything). The set of rules contains "compound-rules", connecting expressions with compounds, and "abstract-rules", connecting expressions with abstracts.

Examples: Let F be an $o(\iota)(\mu)$-object and G an $\iota(\mu)$-object. Let **F** and **G** be words (phrases) of English (Czech) denoting F and G, respectively. Then the following simple compound-rule can be formulated:

Any construction of the form
(f) $\lambda w(F(w)(G(w)))$
is expressed as follows:

English:	**G** is an F,	if **F** is a common noun group,
	G is F,	if **F** is an adjective,
	G F (3rd pers. sg.),	if **F** is a verb;
Czech:	**G** je F,	if **F** is a common noun group or an adjective
	G F (3rd pers. sg.),	if **F** is a verb.

Thus let F be the property of being a smoker, let G be the individual concept of the (present) director of the Škoda works. Let **F** be in English the word "smoker", in Czech the word "kuřák", and **G** in English "the director of the Škoda works", in Czech "ředitel Škodovky".

Then our rule works as follows: The construction (f) is expressed in English:
The director of Škoda works is a smoker.
In Czech:

Ředitel Škodovky je kuřák.

Another rule might be formulated as follows:

Let $\lambda w(\mathrm{P})$ and $\lambda w(\mathrm{Q})$ be linguistic $o(\mu)$-constructions, let C be an $o(o,o)$-object (i.e., a logical connective). Let **P**, **Q** and **C** be the expressions of English (Czech) expressing $\lambda w(\mathrm{P})$, $\lambda w(\mathrm{Q})$, C, respectively. Then for English (Czech) the construction

$\lambda w(C(\mathrm{P},\mathrm{Q}))$ (i.e. the reduct of the $0-1-1$-compound of C, $\lambda w(\mathrm{P})$, $\lambda w(\mathrm{Q})$) is expressed as follows:

PCQ.

Example: Let C be $\wedge$ (conjunction), let $\lambda \mathrm{w}(\mathrm{P})$ be the construction

$\lambda w(\mathrm{O}(w)(\mathrm{A},\mathrm{B}))$

and $\lambda w(\mathrm{Q})$ the construction

$\lambda w(F(w)(B))$

(cf. the preceding examples).

Let **P** be the sentence

A is older than B (*A je starší než B*),

Q the sentence

B is a smoker (*B je kuřák*),

and **C** the word

and (a).

According to our rule, the construction

$\lambda w(\wedge(O(w)(A,B),F(w)(B)))$

is expressed by

A is older than B and B is a smoker

(*A je starší než B a B je kuřák*).

Chapter V

TEMPORAL FACTOR

Now we shall show that an EB must be wider than an MEB to become an adequate base for a linguistic analysis. Let us have the sentence

(1) *Charles was a smoker and he is not a smoker.*

(1) is transformable to

(1′) *Charles was a smoker and Charles is not a smoker.*

Let Ch be the individual expressed (= denoted) by "Charles". Let F be the property of being a smoker, let "and" and "not" denote $\wedge$ (conjunction) and $\sim$ (negation), respectively. Over an MEB, we cannot but analyze (1′) as follows:

(1″) $\lambda w(\wedge(F(w)(Ch),\sim(F(w)(Ch))))$,

which is a reduct of a linguistic construction. (1″) is, however, a construction constructing contradiction, i.e. a proposition that is false in any possible world.

Asking what this absurdity has been caused by, we can see that understanding (1) as a sentence which denotes a non-contradictory proposition is caused by the time factor, i.e. by the fact that the tenses of the two clauses of (1) are different.

A semantic analysis only based on an MEB cannot take the time factor and, consequently, the tenses into account. Nevertheless, there are numerous possibilities of extending an MEB. One of them consists in including into EB the set of "time moments". Bearing in mind some additional extensions (including "space points", etc.), we define an EB_t as any such collection of mutually disjoint non-empty sets as contains an MEB and the set σ_1 of time moments.

Our intuitions connected with an EB_t are the following: in most cases, where A is an intension of the k-th order, $k \geq 1$, i.e. an α^k-object, its "revised" version A' becomes a function from σ_1 into α^k in an EB_t. This corresponds with the fact that any possible world can be conceived of not as a momentaneous distribution of empirical traits but as a set of such distributions possibly different at different time moments. Thus the property of being a smoker can give — as its value in the possible world W at the time moment t_1 — a class of individuals different from that in the world W at the time moment t_2: the people become smokers and cease to be smokers in the same world (e. g., in the actual one). Thus where F as the property of being a smoker was (over an MEB) an $o(\iota)(\mu)$-object, the revised (time dependent, or: t-) property F_t is an $o(\iota)(\mu)(\sigma_1)$-object (over an EB_t).

Similarly, any $o(\mu)$-construction over an MEB becomes an $o(\mu)(\sigma_1)$-construction over an EB_t. Such an $o(\mu)(\sigma_1)$-construction constructs a "t-proposition".

There is also an alternative approach which seems, however, to be more appropriate than the one offered above. We may conceive of the intensions as $\alpha(\sigma_1)(\mu)$-objects where α is a type. This means that, e. g., the property denoted by the word "smoker" would associate every possible world with the "history" of this property so that a concrete class would be the result not only of applying this property to a possible world, but also of applying the result of this application (i.e., an $o(\iota)(\sigma_1)$-object) to a time moment.

Our following examples presuppose the latter approach has been accepted.

The members of σ_1 can be ordered by the relations $<$ and $\leq$. We can define the $o(o(\sigma_1))(\sigma_1)$-objects *Pret* and *Fut* as follows:

Let s, s′ be σ_1-variables. Let A be an $o(\sigma_1)(\mu)$-construction. Using the standard notation of quantifiers and connectives, we can say that the constructions

$$\mathit{Pret}(s)(\mathrm{A}(w)), \qquad \mathit{Fut}(s)(\mathrm{A}(w))$$

are equivalent to

$$\exists s'(<(s',s) \wedge \mathrm{A}(w)(s')), \quad \exists s'(>(s',s) \wedge \mathrm{A}(w)(s')),$$

respectively.

Thus (1) will be analyzed as follows:

$$\lambda w(\lambda s(\wedge(\mathit{Pret}(s)(\lambda s(F(w)(s)(Ch))), \sim(F(w)(s)(Ch))))),$$

which can be modified — according to our definition of Pret and the logical laws of λ-conversion — as

$$\lambda w(\lambda s(\wedge(\exists s'(\wedge(<(s',s), \mathrm{F}(w)(s')(Ch))), \sim(\mathrm{F}(w)(s)(Ch))))).$$

Immediately we can see that these last constructions construct no contradictory proposition (t-proposition).

There are some problems with the time factor that are not directly connected with tenses. But none of them seems to be unsolvable within our theory.

(What has been said about time up to now has been inspired by some remarks made by Tichý in his letters.)

As for the set of space points (σ_2), we need it in order to be able to analyze expressions containing local adverbs and/or local prepositions. Let EB$_{t,l}$ contain an MEB, σ_1 and σ_2. It seems that if a verb denotes an $o(\beta_1, \ldots, \beta_m)(\mu)$-object over an MEB, then it denotes an $o(\beta_1, \ldots, \beta_m)(\sigma_1)(\mu)$-object over an EB$_t$ and it can (not necessarily) denote an $o(\beta_1, \ldots, \beta_m, \sigma_2)(\sigma_1)(\mu)$- or an $o(\beta_1, \ldots, \beta_m, o(\sigma_2))(\sigma_1)(\mu)$-object over an EB$_{t,l}$.

Thus let P be Prague (type: ι), IN be an $o(\sigma_2)(\iota)(\sigma_1)(\mu)$-object (denoted by the preposition "in" in English and "v(e)" in Czech) and S an $o(\iota)(\sigma_1)(\mu)$-object (the *t*-property of sleeping); further let S' be an $o(\iota, o(\sigma_2))(\sigma_1)(\mu)$-object (the t-relation-in-intension between a sleeping individual and the place where this individual sleeps). To use S is sufficient when our task is to analyze, e. g., the sentence

(2) *Charles is sleeping.*

whereas S′ must be used if we wish to analyze the sentence

(3) *Charles sleeps in Prague.*

The analyses of (2) and (3):

(2′) $\lambda w(\lambda s(\mathrm{S}(w)(s)(Ch)))$;

(3′) $\lambda w(\lambda s(\mathrm{S}'(w)(s)(Ch, IN(w)(s)(P))))$.

Clearly, our definitions of linguistic constructions must be modified so as to become definitions of *t*-linguistic or *t*-*l*-linguistic constructions.

Similarly, it is thinkable to extend an MEB or an EB$_t$ or an EB$_{t,l}$ by adding the set, say, ν of natural numbers to the sets o, ι, μ, σ_1, σ_2. The collection EB$_{t,l,n}$ obtained in this way would be appropriate in case we should like to deal with such part of a natural language as contains means for expressing mathematical constructions.

Chapter VI

INDEXICALITY

The investigation of indexical expressions (indexicals) is often considered to be the principal task of pragmatics. (Cf. Montague 1968.) Within our theory, indexicals represent the sphere of external pragmatics, which is to be clearly distinguished from internal pragmatics (see Ch. VII) on the one hand and semantics on the other. Have the following sentence:

(O) *I am hungry now.*

As an expression of a natural language the sentence (O) is not fully analyzable within semantics. Some of its components make it impossible for us to determine which proposition (O) denotes. We should rather say that there is a great number of propositions offering themselves as candidates for the object which (O) is about. The best-suited representative of such a set of propositions seems to be an open construction, i.e., a construction containing free variables (in our case, one of them standing for "I" and another for "now").

We can see that some expressions of a natural language are "dead" in that they do not name any definite object over an EB. Some of them remain to play this role for ever (e. g., syncategorematic expressions). Yet some of them "come to life" when uttered in a concrete situation. Studying utterances instead of sentences, i.e. events instead of abstractions, we study the spatiotemporal circumstances that — together with the sentence itself — determine which proposition is named by uttering the sentence in the very act of communication. Whereas semantics is concerned with propositions, external pragmatics is concerned with associating situations with propositions. No semantic analysis can, of course, determine the object named by "I" (or by "now") in (O). Such expressions are ostensive in that their communicative role is bound to concrete circumstances of the act of uttering an expresion the components of which they are.

It is to be emphasized that the above explanation is not meant to justify any kind of fusion of semantics and pragmatics. On the contrary, just since the role of indexicals clearly differs from that performed by the other expressions, we must carefully distinguish between the semantic analysis on the one hand and the theory of "situation-proposition"-relations (or the theory of indexicals) on the other. In some respects, semantics has priority over pragmatics: What a sentence uttered *hic et nunc* is about (i.e. which proposition it denotes) cannot be established without our knowing the semantics of the given language; but (a) we can learn what a sentence just uttered is about without knowing the circumstances of the act of utterance in case the sentence in question does not contain any indexicals, and (b) even if the sentence does contain some indexicals, we can, on the basis of the pure semantics, establish at least the scheme of possible $o(\mu)$-constructions expressed by this clause.

Let us return to what has been said at the beginning of the present chapter. Take the following sentence:

(1) *I am hungry.*

In the vocabulary of English satisfying the principles of our theory we can find that "to be hungry" names an $o(\iota)(\sigma_1)(\mu)$-atom (say, H) and "I" is an indexical word that can name an individual. As an indexical word, however, it denotes no concrete individual.

Let A,B, C, ... be members of the universe in question. Using (1) in particular situations, we can express by it the following constructions:

(1_1) $\lambda w(\lambda s(H(w)(s)(A)))$;

(1_2) $\lambda w(\lambda s(H(w)(s)(B)))$;

(1_3) $\lambda w(\lambda s(H(w)(s)(C)))$;

. .

. .

. .

Thus using (1) in situations S_1, S_2, S_3, ..., we simultaneously use a function that associates "I" with various individuals dependently on S_1, S_2, S_3, ... This function, say F_I, is not what can be called a semantic function: its domain is the set of situations, and, therefore, such a function is a pragmatic function. Associating "I" with the individuals A, B, C, ..., F_I thereby associates the sentence (1) with the constructions (1_1), (1_2), (1_3), ..., and, consequently, with different propositions.

Now, have the open construction

$(1)'$ $\lambda w(\lambda s(\mathrm{H}(w)(s)(x)))$,

where x is an ι-variable. If x v-constructs A, v′-constructs B, v″-constructs C, ..., then the v-instance of $(1)'$ is (1_1), the v′-instance of $(1)'$ is (1_2), the v″-instance of $(1)'$ is (1_3), ... Therefore, we could conceive of F_I as a principle that determines — dependently on situations — which valuation should be selected as regards x in $(1)'$ if $(1)'$ is associated with (1). Similarly, let F_{HE} be another function, such as associates the situations with individuals and is used whenever we use the English word "he" (or, the Czech word "on", etc.). Then F_{HE} will play the role of a principle that determines which valuation should be selected as regards x in $(1)'$ if $(1)'$ is associated with (2):

(2) *He is hungry.*

Our example can be generalized for all the cases in which the sentence in question contains some indexical words (as "we", "now", "this", "here", etc.). Such sentences (let us call them" indexical sentences" or simply "i-sentences") are able to express a construction whenever the situation determines what entities are meant by the various indexical words. These entities may be of various types; in case of "I", "he", "it", etc., they are ι-objects, in case of "we", "they", etc., they are $o(\iota)$-objects, in case of "now", etc., they are σ_1 or $o(\sigma_1)$-objects, etc., etc.

Ours is the following question: How to analyze an i-sentence?

Any adequate answer to our question must take into account that an i-sentence does not determine a proposition unless there is given a concrete situation of use. Thus without pragmatic determination no analysis of an i-sentence is possible, which amounts to saying that without pragmatics it is impossible for us to learn what is meant by such an i-sentence.

Yet, something is given if an "isolated" i-sentence is to be dealt with: where $\mathrm{i}_1, \ldots, \mathrm{i}_n$ are the indexical expressions contained in the sentence **A**, we can associate the sentence **A** with such an open construction A such as contains variables $x_{\mathrm{i}_1}, \ldots, x_{\mathrm{i}_n}$ in place of those atoms which would respectively be named by $\mathrm{i}_1, \ldots, \mathrm{i}_n$ if $\mathrm{i}_1, \ldots, \mathrm{i}_n$ were not indexical. Besides, we can put the pragmatic functions $\mathrm{F}_{\mathrm{i}_1}, \ldots, \mathrm{F}_{\mathrm{i}_n}$ within brackets behind the variables. Thus the sentence

(3) *He is hungry now.*

would be associated with the following construction:

$$(3') \qquad \lambda w(\mathrm{H}(w)(\mathrm{s})[\mathrm{F}_{\mathrm{NOW}}](\mathrm{x}[\mathrm{F}_{\mathrm{HE}}]))$$

where s is a σ_1-variable and x an ι-variable.

A situation may be conceived of as an n-tuple consisting of a time moment t, the place, the speaker m, the hearer h, the objects that are spoken about, etc. Given a situation S containing the time moment T and the individual K that is being spoken about, we have

$$\mathrm{F}_{\mathrm{NOW}}(\mathrm{S}) = T, \qquad \mathrm{F}_{\mathrm{HE}}(\mathrm{S}) = K$$

and the relevant "pragmatic instance" of $(3')$ is

$$(3'') \qquad \lambda w(\mathrm{H}(w)(T)(K)).$$

Now we can say that (3) expresses $(3'')$ in the situation S. If no situation is given, (3) expresses no construction, but it is associated with $(3')$. This reflects the fact that (3) qua a "dead sentence" names no specified proposition.

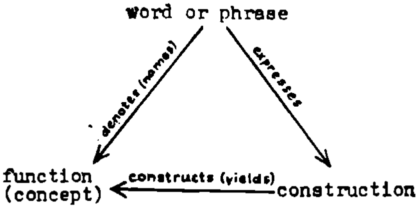
If we want to analyze, e. g., English sentences, we can choose one of, at least, two approaches: we can either refuse to analyze any i-sentence and demand that all indexical "parameters" be substituted for by non-indexical expressions, or we can associate such an i-sentence with an open construction in the above way. In the latter case, however, we do not say that the sentence in question expresses the respective open construction; it does not express any construction at all.

Remark: The present conception basically differs from that advocated by Montague, D. Scott, etc., in that we do not mix together possible worlds and situations; semantics (possible worlds!) and pragmatics (situations!) should be kept apart — the concern of the former is different from that of the latter.

Chapter VII

THE SEMANTIC TRIANGLE AND PRAGMATICS

This chapter is meant to illustrate the relation between semantics as presented above and pragmatics. The most suitable starting point for graphical illustration seems to be the following triangle:



A word or a phrase expresses a certain construction, which in turn constructs (or, yields) the respective function (in our case, a concept). This function is what the given word or phrase denotes (or, names). The denoting (naming) relation between words (phrases) and functions is established through the mediation of constructions. Functions (concepts) are intensions; hence words and phrases denote intensions. Extensions, i.e. individuals, classes of individuals, relations between individuals, relations between classes of individuals, etc., are in our cases the values of the functions in questions. There is only one case in which a function is both an intension and an extension: the nullary function. The value of a nullary function is the function itself. Words and phrases denoting nullary functions are called proper names. Proper names are the only cases in which language expressions denote extensions. Thus we know what proper names speak about if we know their extensions, i. e., if we know precisely which individual, class of individuals, etc., of our universe of discourse is denoted by a given proper name. As has been shown before, this is not the case with other words and phrases denoting non-nullary functions. We know what such an expression speaks about because we know, or can construct, the respective function (concept), which may be conceived of as instructions of how to look for the values of this function in different possible worlds. Whether we find the value in a given (e.g., the actual) world or not is by no means the condition of understanding the given expression.

Similar to words and phrases, sentences or clauses (further only sentences) also express constructions and denote (name) functions. The construction expressed by a sentence contains all the constructions expressed by the separate words and phrases in the given sentence as well as their mutual arrangement. Such a construction yields a function which is, of course, not independent of the functions denoted by the separate words and phrases of the sentence, but which — at the same time — is not a mere collection of the separate functions either. The construction expressed by a sentence yields a function from possible worlds into truth-values, i. e. a function that assigns **a** truth-value to any possible world in which it is defined. This kind of function is called a proposition. In contradistinction to Frege (cf. Frege 1892), a sentence expresses, **a** construction and denotes a proposition:

sentence

f

propositions : **construction**

What has happened to Frege's truth-values? They simply represent the values of the proposition in the possible worlds, i. e., the extensions, which are not directly denoted by the sentence.

To understand a sentence means to know what it denotes; it means to know the respective proposition, the function from possible worlds into truth-values. To know this function is tantamount to knowing the way of how to look for the truth-value of the sentence in any possible (i.e. also the actual) world. We know what the sentence

The President of Czechoslovakia opened the Championship

denotes and we understand it, not because we know whether it is true or false, but because we know the functions (concepts) denoted by "the President of Czechoslovakia", "opened", "the Championship", and we also know the proposition, i.e., how to verify the sentence in any possible world. If it is true in the actual world, we say that it is a fact; but fact or no fact, it has no bearing whatsoever on our understanding the sentence.

In the same way as we understand "Pegasus", "mermaid" and other mythological and fairy-tale expressions without finding the values of their respective functions (i.e. the occupants of their offices) in the actual world, we also understand sentences without finding the truth-values of "their" propositions in the actual world. In addition to that, by using such sentences in a way suggesting that "their" propositions are true in some possible world,, we are in fact building up or "discovering" the subset of possible worlds in which these propositions are true. This is not only the case of fairy tales, the same holds good for scientific theories. We understand them because we know what they denote, we know the concepts and propositions. Furthermore we are entitled to assume that the propositions are true in some possible worlds. Hence to verify a theory means to find out whether the actual world is among them, i.e., to find out whether all the values of the propositions in the actual world are "truth".

What is meant by "using sentences in a way suggesting that 'their' proposi-