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LOGICAL ANALYSIS OF NATURAL LANGUAGE AS AN ORGANIC PART OF LOGIC

Two kinds of logical errors

Let an argument be of the form $S_1, \ldots S_m \Rightarrow D$, where S_i are premises and D is the conclusion. If there is such a valuation of the free variables that replace extra-logical terms in the premises where all S_i are true while D is false we have committed an error of the first kind.

Example
If the contract has been signed then the building continues to be owned by XY
<u>*The contract has not been signed*</u>
Thus the building does not continue to be owned by XY

What happened: The conclusion has been derived according to an invalid scheme

$$A \supset B, \neg A \Rightarrow \neg B.$$

Similar examples can be easily found when the premises and conclusion possess structures explicated in 1st order predicate logic.
But what can be said about the following example:

The Czech President is the husband of XY ……… true
<u>*Charles wants to be the Czech President* ……… true</u>
Charles wants to be the husband of XY ……… false

It is not very difficult to say that the conclusion does not follow from premises. Yet which invalid logical scheme has been used?
The only rule that can be made responsible for this argument is evidently Leibniz's rule of *Indiscernibility of identicals,* i.e.,

$$a = b\ ,\ \Phi(\ldots a \ldots)\ \Rightarrow \Phi(\ldots b \ldots)$$

Indeed, taking *The Czech President* for *a, the husband of XY* for *b* and *Charles wants to be* for the context Φ we get justification of the conclusion due to Leibniz.

The problem is well-known. We know some attempts to solve it, and P. Tichý, when showing that his transparent intensional logic (TIL) offers an optimum solution, says:

> *It turns out that on this approach* (i.e., TIL) *there is no need to say (as Frege does) that the descriptive terms are referentially ambiguous or to deny (as Russell does) that descriptive terms represent self-contained units of meaning. There is also no need to tolerate (as Montague does) exceptions to the Principle of Functionality.*[1]

Before showing the core of the TIL solution let us emphasize one point which is frequently underestimated:

The errors of the second kind consist in mechanically applying some logical rules to the text of the given task *without performing first a logical analysis of the text that is given as an expression of a natural language*. Or perhaps such an analysis is very superficial and, therefore, simplifying.

In our case, we were satisfied with stating that *the Czech President* (or *the President of Czech Republic*) is a term as well as *the husband of XY* and the context is simply the sentence *Charles wants to be a / b*. It seems as if such an 'analysis' were sufficient for applying Leibniz. Since however Leibniz rule is evidently sound and our application thereof was connected with an invalid argument we have to find out *why the rule cannot be applied.*

It seems that the most popular scenario of the solution is Frege's proposal from his[2] known as 'reference shift'. We will show why his way out cannot be accepted.

Frege's solution

Frege's solution is well-known, so we will be rather brief.

First of all, we can understand why his attempt seems to be welcome in our situation: Leibniz (or any valid logical rule) can be shown to be

1 Pavel Tichý, The indiscernibility of individuals, *Studia Logica*, 1986, vol. 45, pp. 251–273: 251, reprinted in P. Tichý, *Collected Papers in Logic and Philosophy*, V. Svoboda – B. Jespersen – C. Cheyne (eds.), Prague: Filosofia – Dunedin: University of Otago Press 2004, pp. 649–671: 649.

2 G. Frege, Über Sinn und Bedeutung, *Zeitschrift für Philosophie und philosophische Kritik*, vol. 100, 1892, pp. 25–50.

inapplicable as soon as a term whose meaning is presupposed to be constant is proved to be ambiguous. Frege's solution seems to offer such an ambiguity of the term *The President of Czech Republic.* Briefly, Frege's argumentation, to be found, e.g., in Frege,[3] see Geach, Black,[4] is based on Frege's introduction of *sense* between expression and 'reference' (Church's term is 'denotation'). The idea itself is positive as the claim that to get a reference of an expression we have to distinguish between this reference and the sense as "the mode of presentation" (of the reference). Frege did not *define* the sense so that the contemporary semantics of natural language had spent much time to do it (below we will show the result offered by TIL) but he soon discovered that some problems arose with keeping the principle of compositionality: In some ('indirect') contexts such as sentences where the subordinate clause begins by some attitudinal verb,

> *it is not permissible to replace one expression in the subordinate clause by another having the same customary reference, but only by one having the same indirect reference, i.e. the same customary sense.*[5]

Now compositionality was saved, because if somebody could ask whether Shakespeare was the author of *Hamlet* and if *the author of Hamlet* 'normally' denoted Shakespeare the question could have been interpreted as asking whether Shakespeare was (given by ? not very clear!) the *sense of* the expression *the author of Hamlet*, rather than asking whether Shakespeare was Shakespeare. Without this 'reference shift' the question would be nonsensical, or so Frege believed.

But even setting aside the absence of the definition of *sense* we have to state that the price was too high. Frege's system became *contextualistic.* It would be impossible to ascribe some semantic value to an expression unless asking some (which one?) context. And it would be enigmatic to decipher semantics in the case when there were more embedded subordinate clauses, like *Charles is convinced that Peter believes that Charles is an idiot.*

So we can say that Frege's solution is no solution: It would imply that *The President of Czech Republic* is an ambiguous term in that it denotes its reference in the first premise while it denotes its 'customary sense' in the second premise. Leibniz cannot be applied since $a \neq b$ this time. But the extreme contextualism is the price.[6]

3 *Ibidem*.

4 P. Geach – M. Black (eds.), *Translations from the Philosophical Writings of Gottlob Frege*, Oxford: Basil Blackwell 1952.

5 *Ibidem*, p. 67.

6 For details see P. Tichý, *The Foundations of Frege's Logic*, Berlin – New York: De Gruyter 1988.

We will show that the problem can be solved so that
a) the principle of compositionality holds,
b) what can be defined as *sense* (or *meaning*) is independent of any context and the same holds about denotation,
c) the solution is not *ad hoc* but is based on a systematic theory.

Systematic solutions vs. *ad hoc* attempts

In M. Duží, B. Jespersen, P. Materna[7] two approaches to logical semantics are confronted: the *bottom-up* and the *top-down* approach. On the former approach logic begins to solve the simplest problems (atomic sentences *Pa*), then truth functions (negation, conjunction,…) are added. Then one can see that such logically relevant expressions like *every(body), some…* have to be added together with predication, and predicate logic(s) are built up. On this level we again find not solvable tasks, and add new logical objects: modalities. The old puzzle with the number of planets demonstrates that something new has to be added:

Necessarily, 8 is greater than 5
<u>*The number of planets is 8*</u>
Necessarily, the number of planets is greater than 5

The modal logicians can explain: the conclusion is not derivable because co-extensional terms cannot be substituted into contexts governed by the modal operator □. But why is this forbidden? No explanation.
The development continues. Why is the following argument invalid?

The murderer is XY
<u>*The detective knows that the murderer bears glasses.*</u>
The detective knows that XY bears glasses.

Attitudes have to be added. Etc.

The 1st order predicate logic is a typical tool of logical analyses. One feature of PL is emphasized in DJM: the logical form does not reveal the semantic character of whose form irt is. Thus the form

7 M. Duží – B. Jespersen – P. Materna, *Procedural Semantics for Hyperintensional Logic,* Berlin: Springer 2010.

$$\exists x\ (P(x) \wedge \mathrm{Q}(x))$$

is the form of *Some primes are even* (an analytically true sentence), *Some even numbers are odd* (an analytically false sentence) as well as of *Some clever students are lazy* (an empirical sentence true dependently on state of the world. Thus semantics comes with *interpretation.*

Some cases are surprising. The following argument seems to be convincing but is simply invalid:

All primes greater than 2 are odd.
The greatest prime is greater than 2
The greatest prime is odd

Here the 'neutral' form is

$$\frac{\forall x\ (P(x) \supset Q(x)) \qquad P(d)}{Q(d)}$$

The interpretation is determined by our understanding the particular expressions. The only true expression here is the first premise. And the second premise as well as the conclusion lacks any truth-value.

This bottom-up approach means that the meaning / reference of an expression differs in various ways of analysis. Anticipating our later analyses we can state that while the meaning (Frege's sense) of an expression from the viewpoint of the top-down approach is always an abstract procedure ("construction") and the denotation (we will use this term instead of "reference") is what the meaning constructs, the bottom-up approach makes meaning and denotation dependent on the way of given analysis. Thus an empirical sentence denotes on every level of analysis a non-trivial proposition for the top-down approach, it will denote a truth-value from the viewpoint of a truth-functional analysis and – maybe – a proposition from the viewpoint of predicate logic (in Montague) within the bottom-up approach. Thus this latter approach has inherited Frege's contextualism.

The hyperintensional solution

Let us have some thoughts considering a characteristic example:

$2 + 3 = (+)\surd\ 25$
Charles calculates $2 + 3$
Charles calculates $(+)\surd\ 25$

This is again the case when the argument is invalid although the Leibniz rule seems to have been applied. For Frege the reason would be that '2 + 3' in the first premise denotes a number while the same expression in the second premise denotes its ('customary') sense so that we can see that this expression is ambiguous and Leibniz cannot be applied.

We have however stated that this solution is untenable because of Frege's contextualism. Meaning and so the denotation of '2 + 3' should be the same in both premises. But then nothing would prevent us form using Leibniz!

We will now just verbally suggest the solution, whereupon we will try to explain this solution in more details.

Let us consider the first premise. The identity looks unbeatable. If we succeeded to show that actually an important interpretation can view the premise as claiming non-identity then one of the arguments for applying Leibniz would drop out. So let us try.

Properly speaking, what does the first premise claim? Let [2 + 3] be the sense of the expression '2 + 3', in general [A] be the sense of the expression A. The first premise can be interpreted as claiming the identity of the denotations of '2 + 3' and '(+)√ 25'. But the denotation of an expression should be what the sense determines. In other words, one interpretation is that what is the result of *executing* [2 + 3] is the same object as what is the result of *executing* ['(+)√ 25']. On this interpretation the first premise is true. But the second premise does not use [2 + 3]: Charles does not relate himself to what is the result of using the sense, i.e. to the denotation, i.e. to the number 5: his calculating activity concerns the sense itself, i.e. the sense is here just *displayed.* Thus if '2 + 3' is used in both premises then the second premise is not true.

On the other hand, if '2 + 3' is displayed in both premises then the first premise is not true. The identity claimed by this premise does *not* say that the sense of '2 + 3' is identical with the sense of '(+)√ 25'. [2 + 3] ≠ [(+)√ 25].

In other words, Leibniz is never applicable.

Now, this solution is likable and satisfies our promise that meaning will be independent of any context. Indeed, what *is* dependent on context is whether the meaning is executed or displayed. Indeed, the meaning of '2 + 3' is the same in both premises, and the semantically relevant distinction is given just by the *executed* / *displayed* distinction.

Yet all this cannot convince anybody that a solid solution has been proposed. Instead of a systematic theory some suspect and rather verbal, meta-theoretical considerations like *the sense is executed, displayed* are offered. A question is therefore justified:

> *What kind of object is the meaning (sense) and how can logic ensure, that this object can be used or mentioned so that exact definitions replaced mere verbal characteristic?*

So an *explication* is needed. This explication can be found in P. Tichý[8] and M. Duží, B. Jespersen, P. Materna [9] and makes up the core of TIL. Naturally, we cannot reproduce the whole explication here, so we will try to say more important theses claimed by TIL and explain the idea that problems with the 'errors of the second kind' can be solved within a *hyperintensional* system (which TIL is).
The main thesis is:

> *The meaning (or: what Frege meant by "sense") of an expression is always an abstract procedure.*

More details:

> *Abstract procedures are explicated in a logic based on functional approach.*
>
> *Functions cannot be procedures because of the principle* $\forall x\ (f(x) \supset g(x)) \supset f = g$.
>
> *Using* λ*-calculus with simple types an expressive intensional logic is built up.*
>
> *Adding a ramified hierarchy of types a hyperintensional logic arises.*
>
> *Abstract procedures are explicated as* constructions*. The main two constructions are Composition (known as Application in* λ*-calculi) and Closure (see Abstraction in* λ*-calculi).*
>
> *Hyperintensionality means that even equivalent constructions are not necessarily identical.*

This is the point that explains why Leibniz cannot be applied if only equivalence holds.

Indeed, if the Fregean sense is a construction, then the denotation (if any) is what the sense *constructs.* Then, of course, identity of what is constructed is distinct from the identity of the constructions themselves. Observe that the problems that led Frege to propose his reference shift are solvable without any shift: *it suffices to distinguish executing and displaying the sense.*

One can ask, of course, how this distinction becomes a part of logic instead of being just cited as a meta-claim. The exact answer would require introducing some more definitions and arguments but here we can suggest the way TIL does it within its ramified hierarchy of types.

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8 *Ibidem*.
9 *Ibidem*.

The inductive definition of constructions contains a most important definition of the construction called *Trivialization:*

Where X is any object including constructions ${}^0X$ is a construction called *Trivialization:* it constructs just X without any change.

The construction defined in this way makes it possible to define a ramified hierarchy of types, where types of higher orders are defined. (Briefly, the constructions get a higher type than what is constructed.) Thus ${}^0X$ is always of a higher type than X. And the *Trivialization* starts this raising of types. Thus if X is an object of a type of order n, the type of ${}^0X$ is of order $n + 1$.

This ramified hierarchy essentially increases the expressivity. It makes it possible to not only execute but also display procedures. Procedures, i.e., constructions make up a special class of objects.

The *hyperintensional (conceptual) contexts* are such contexts where *constructions are displayed.* The *functional contexts* are contexts where constructions are *executed* to construct functions (*intensional level*) or values of functions on arguments (*extensional level*). The deductive rules have to be adapted to the kind of context (level). Thus Leibniz's $a = b$ means identity of constructions or of functions or of the values of functions. The difference between the intensional and the extensional level is known (when analyzing empirical expressions) as the difference between *de dicto* and *de re.*[10]

As an example of analyzing premises we adduce our example

$$\begin{array}{c} 2 + 3 = (+)\surd\ 25 \\ \underline{\text{Charles calculates } 2 + 3} \\ \text{Charles calculates } (+)\surd\ 25 \end{array}$$

We cannot explain the details of our analysis but the result should be clear: the conditions of applicability Leibniz are not satisfied.

$$\begin{array}{c} [{}^0{=}\ [{}^0{+}\ {}^0 2\ {}^0 3]\ [{}^0(+)\surd\ 25]] \\ \underline{[{}^0 Calc_{wt}\ {}^0 Charles\ {}^0[{}^0{+}\ {}^0 2\ {}^0 3]]} \\ \text{Leibniz not applicable} \end{array}$$

Leibniz would be formally applicable if either

$$[{}^0{=}\ {}^0[{}^0{+}\ {}^0 2\ {}^0 3]\ {}^0[{}^0(+)\surd\ 25]]$$

or

$$[{}^0 Calc_{wt}\ {}^0 Charles\ [{}^0{+}\ {}^0 2\ {}^0 3]]$$

10 See *ibidem*, e.g. 233.

held true but in the first case the first premise would be false, in the second case the second premise could not be true (Charles does not "calculate 5").

We have shown that the inapplicability of Leibniz can be exactly justified without giving up compositionality and without accepting contextualist explanation. Hopefully we can see that the solution is the result of a broader conception and that it is no *ad hoc solution:* the hyperintensional system of TIL has not been created just in order to solve inapplicability of Leibniz rule in some cases.

But on the occasion of solving some similar puzzles we can claim that among the *logical aspects of rational agency* we find understanding natural language expressions by means of their *attentive logical analysis*.

Gamut, Cresswell

It is especially the problem of analyzing propositional and notional attitudes where the insufficiency of 1st order systems emerged. In an good overview of (especially Montagovian) intensional logic (the collective) L. T. F. Gamut[11] speaks about the necessity to do "a more refined intensional semantics" because some examples "indicate that more than just logical equivalence, that is to say, equality of intension, is required for interchangeability salva veritate in hyperintensional contexts."[12] Gamut further aptly suspects: "Perhaps the ways in which the intensions of expressions are built up from intensions of their composite parts should also be taken into account."[13]

This characteristic of hyperintensionality has been formulated in 1991. It is too general and the author admits that he is not very specific. Therefore he proposes that "semantics must join forces with pragmatics in order to give an adequate treatment of hyperintensional contexts".[14] The system we have briefly characterized has shown that semantics can offer such an adequate treatment not joining forces with pragmatics.

Surprisingly Gamut has not quoted Cresswell's[15] where an attempt at semantically defining hyperintensionality is made. It can be shown, however, that neither Cresswell's attempt can be accepted: his reducing the way a meaning is structured to *n*-tuples does not realize the important step to higher-order types. He says…

11 L. T. F. Gamut, *Logic, Language and Meaning*, vol. II, Chicago – London: The University of Chicago Press 1991.

12 *Ibidem*, p. 73.

13 *Ibidem*.

14 *Ibidem*.

15 M. J, Cresswell, Hyperintensional logic, *Studia Logica*, 1975, vol. 34, pp. 25–38.

> *The meaning…is simply the n + 1 tuple consisting of the meaning of the functor together with the meanings of its arguments.*[16]

On p. 32 he summarizes: "Truth-conditional semantics is sufficient to determine meaning."

Thus we get a set-theoretical paradigm: *What counts is always the result of applying a procedure, rather than the procedure itself.*

It seems that the best way how to explicate hyperintensionality consists in accepting the procedural view.[17]

Přetištěno z mimořádného čísla Newsletteru FÚ AV ČR.

16 *Ibidem*, p. 30.

17 A nice summarization of this view can be found in Y. N. Moschovakis' title of one of his papers Sense and denotation as algorithm and value, in J. Väänänen – J. Oikkonen (eds.), *Lecture Notes in Logic*, vol. 2, Berlin: Springer 1994, pp. 210–249.

ABSTRAKT

LOGICKÁ ANALÝZA PŘIROZENÉHO JAZYKA JAKOŽTO ORGANICKÁ SOUČÁST LOGIKY

Existují dva druhy logických chyb. Buď je užito neplatné schéma argumentace, nebo je naše analýza premis chybná. Žádná extenzionální ani intenzionální teorie nemůže vyřešit následující problém spojený s analýzou výrazů přirozeného jazyka: Leibnizův princip substituce identického za identické obsahuje podmínku $a = b$. Jak extenzionální, tak i intenzionální systémy (alespoň jsou-li intenze definovány jako funkce z možných světů) se při analýze této podmínky formulované v přirozeném jazyce spokojí s tím, je-li a nahodile nebo logicky či analyticky ekvivalentní s b, zatímco to nemusí stačit pro aplikaci Leibnizova principu. Uvádíme příklady, ukazující, jak je v takových případech absurdní aplikovat tento princip. Je myslitelná následující náprava: mohli bychom se pokusit formulovat nějaké axiómy nebo snad metasystémová pravidla, která by eliminovala kritické případy. To by však znamenalo, že vzniká nová teorie jen proto, aby bylo zabráněno nesprávné aplikaci Leibnizova pravidla. Místo toho nabízíme *procedurální* analýzu (výrazů) přirozeného jazyka, která umožní určit jednoznačně jejich smysl a tedy i denotaci tak, že zmíněné kritické případy nemohou nastat. Ukážeme, že hyperintenzionální systém, který definuje Transparentní intenzionální logika, dokáže obecně řešit hlavní problémy spojené s užitím výrazů přirozeného jazyka.

Klíčová slova: Transparentní intenzionální logika, procedurální analýza, Leibnizův princip

SUMMARY

LOGICAL ANALYSIS OF NATURAL LANGUAGE AS AN ORGANIC PART OF LOGIC

There are two kinds of logical errors. Either you use a non-valid scheme of an argument or your analysis of the premises is mistaken. No extensional or intensional theory can solve the following problem connected with analyzing NL expressions: The Leibniz principle of substituting identical for identical contains the condition $a = b$. Extensional as well as intensional systems (at least if intensions are defined as functions from possible worlds) analyzing this condition as formulated in natural language are happy if a is contingently or logically or analytically equivalent with b, while this may be insufficient for applying Leibniz. Examples are adduced that show the absurdity of applying Leibniz in such cases. A following remedy is thinkable: one could try to formulate some axioms or perhaps meta-formulated rules that would eliminate the critical cases. This would mean however that a new theory came into being just to shield us from incorrectly applying Leibniz rule. Instead a *procedural* analysis of NL expressions is offered that makes it possible to unambiguously determine their sense and so their denotation in such a way that the above mentioned critical cases cannot set in. It is shown that the hyperintensional system defined by Transparent intensional logic is able to generally solve the main problems connected with using NL expressions.

Key words: Transparent intensional logic, procedural analysis, Leibniz principle

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