Sancar, Esmanur; Özkaya, Merve

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STUDY

ERROR-BASED ACTIVITY APPLICATIONS IN SIXTH GRADE FRACTION TEACHING

Esmanur Sancar[a], Merve Özkaya[b]

[a] Fatih Faculty of Education, Trabzon University, Turkey
[b] Kazım Karabekir Faculty of Education, Atatürk University, Turkey

ABSTRACT

This study aims to determine how the error-based activity (EBA) applications used in teaching fractions to sixth-grade students affect the academic achievement of the students, the perceived error climate in the classroom, student math anxiety levels, the permanence of the knowledge, and student views on the EBA applications. In the quantitative part of this research, a mixed-method study, a quasi-experimental design with a pretest-posttest control group was used to examine the effects of EBA applications on math achievement, math anxiety levels, classroom error climate, and permanence. A purposive sampling method was used to collect the quantitative data for the study. In the qualitative part of this research, a case study, semi-structured interviews were conducted with nine volunteers from the experimental group to determine the student views of EBA practices. Analyzing the data obtained showed that EBA had a positive effect on student success, positively affected the perceived error climate in the classroom, and effectively reduced student math anxiety levels and made the knowledge permanent. Apart from this, the results indicated that EBA applications positively affect students in terms of cognitive and affective aspects, increase the quality of teaching, and can be effective in different subjects and courses.

KEYWORDS

error-based activity; classroom error climate; math anxiety; error orientation; fractions

CORRESPONDING AUTHOR
Merve Özkaya, Department of Mathematics and Science Education, Kazım Karabekir Faculty of Education, Atatürk University, 25240 Erzurum, Turkey
e-mail: mdurkaya@atauni.edu.tr

Introduction

All students make some mistakes in mathematics lessons. Unsuccessful students make mistakes often. Successful students also sometimes struggle with new easy topics and make mistakes. But this is expected (Hodgen & Askew, 2010). Unfortunately, many students are afraid of making mistakes due to their personal characteristics (Käfer et al., 2018; Tulis et al., 2016). Without a positive attitude toward mistakes, learning does not take place (NCTM, 2000). Errors must be incorporated into the learning environment in the right way or students will believe that they have failed (Matteucci et al., 2015). Although making mistakes is seen as a negative situation, mistakes can often be more informative than correct answers in the learning environment (Kurdal, 2016), because the mistake made by the student gives the educator the opportunity to determine the extent to which the subject is understood and to provide feedback to the student (Hodgen & Askew, 2010).

Since mistakes cannot be prevented, suitable environments can be created for better recognition of mistakes and learning from mistakes. These mistakes should be used appropriately, discussed in the course, and turned into opportunities (Borasi, 1988, 1996). Students should be given the opportunity to examine and criticize different ways of thinking. Thus, errors should be used as a teaching tool in the classroom and students should be supported in their creation of knowledge by improving their understanding (Borasi, 1994; Heinze, 2005; Kurdal, 2016; Seifried & Wuttke, 2010). Große and Renkl (2007) referred to this as learning with incorrect solutions. Learning with incorrect solutions is important because being able to explain why the incorrect is wrong as well as saying why the correct is right is seen as proof that learning has been effective (Siegler, 2002). Since we use learning with incorrect solutions as a method in our study and have also included some more conceptual errors, we refer to the activities as error-based activities (EBA).

1 Theoretical framework

1.1 Positive and Negative Knowledge

Mistakes have been seen as obstacles for students to learning the truth (Ingram et al., 2015; Matteucci et al., 2015; Metcalfe, 2017; Swan, 2002; Wasilewski, 2023). Especially toward the end of the 20th century, when the effects of the constructivist approach emerged in education, mistakes became an important part of learning (Konyalıoğlu et al., 2019; Oser & Spychiger, 2005). Thus, the concept of negative knowledge was formed. Negative knowledge is when invalid or unnecessary knowledge is not seen as worthless on the way to the goal of completing a specific task. In short, negative knowledge is when an individual knows what is wrong and what should not be done about a particular task (Gartmeier et al., 2008). In the field of education, Oser and Spychiger (2005) evaluated negative knowledge as a part of learning and put forward the negative knowledge theory, which states that negative knowledge plays an important role in preventing errors. According to the theory, errors in mathematics lessons significantly support the development of mathematical knowledge (Heinze, 2005). This theory also mentioned positive knowledge.

Positive knowledge, free from errors, is defined as knowing what a person should do or avoiding mistakes (Akpınar & Akdoğan, 2010). In short, it is doing the right thing (Spychiger et al., 2006). Using only positive knowledge, however, the student continues to think in previous ways and is unable to use new ways of thinking. For this reason, the cognitive approach, which advocates the use of only positive knowledge, and the behavioral approach, which ignores errors, have been criticized (Heinze, 2005; Parviainen & Eriksson, 2006). Positive knowledge alone cannot enable a metacognitive approach; teaching with activities aimed at gaining negative knowledge contributes to metacognitive development (Melis et al., 2010). Although positive knowledge is necessary for learning, it is not sufficient on its own. According to Heinze (2005), when making mistakes is prohibited during the learning process, knowledge cannot be structured. Negative knowledge is needed to eliminate this inadequacy. In this context, instead of saying that one of these two types of knowledge is more valuable than the other, it would be more appropriate to use these two types of knowledge together and consider negative knowledge, which supports cognitive and affective features, as a complement to positive knowledge. Positive knowledge is lacking in terms of questioning and gaining different perspectives. Therefore, it is thought that negative and positive knowledge should be used together in learning environments (Akpınar & Akdoğan, 2010; Heinze, 2005).

1.2 Error Based Activities

Positive knowledge is necessary for learning, but it is not sufficient. Negative knowledge is needed to complete the missing points of positive knowledge. In order to meet this need, a constructivist approach should be adopted. In this approach, correct knowledge can be clearly determined by using errors, an environment can be created for high-level learning, and questioning, discovery, and knowledge structuring can be achieved. Thus, by appropriately incorporating errors into the learning process, the learning action can be conducted correctly (Gedik, 2014; Heinze 2005; Özkaya, 2015; Özkaya et al., 2022).

In the learning environment, mistakes are seen as a springboard (Borasi, 1994). In other words, students can turn mistakes into useful steps in their learning process (Borasi, 1988; Heinze & Reiss, 2007; Rach et al., 2013). Some studies have achieved positive results in relation to the use of errors in the classroom (Abay & Clores, 2022; Durkin & Rittle-Johnson, 2012; Heemsoth & Heinze, 2016; Heinze & Reiss, 2007; Matteucci et al., 2015; Rach et al., 2013). But first, the students' emotional-motivational positioning toward mistakes needs to be improved to develop a positive understanding of errors in learning (Grassinger & Dresel, 2017).

When students make mistakes in lessons, they often develop a negative emotional-motivational state (Tulis et al., 2016). Teachers, in particular, should make an effort to correct this situation. According to Tulis (2013), teacher behavior toward mistakes affects student performance in learning from mistakes. Teacher behaviors toward errors include organizing and implementing activities to reveal errors, deliberately addressing errors, promoting conceptual learning by discussing errors, and mobilizing students toward errors (Bray & Santagata, 2013). An important point here is to deliberately turn to mistakes. Teachers must be able to deliberately produce inaccuracies and, of course, be willing to do so. Palkki and Hastö (2019) stated that even when teachers had developed a positive perspective on mistakes, they were not sure about the effectiveness of mistakes in learning. However, making errors is an important experience that increases learning and motivation (McMillan & Moore, 2020). Dresel et al. (2013) explained this experience as the ability to transform errors into action processes. Students who can transform errors into action processes explain the underlying cause of the error with cognitive and metacognitive processes, thereby gaining affective abilities such as resilience and risk-taking (Dresel et al., 2013; McMillan & Moore, 2020).

It is clear that mistakes benefit the student cognitively, because the act of learning is associated with detecting errors and discovering their causes. If learning has occurred, errors must be identified with the knowledge learned (Konyalıoğlu et al., 2010). Errors play a key role in understanding why and how wrong situations occur. In this way, learning negative knowledge is effective (Dalehefte et al., 2012). Mistakes are an effective tool for revealing the learning difficulties experienced by the student and in reducing and improving these difficulties. By using errors appropriately, students can become aware of their mistakes so that they are not deprived of the opportunity to explain and correct them. A difficult lesson can become extremely motivating thanks to activities in which mistakes are used as instruction (Borasi, 1987). Errors in the classroom are analyzed by teachers in different ways. Peng (2010) mentioned that the types of error analysis, defined as identification, interpretation, evaluation, and correction, can be developed from the definition at the bottom to the correction at the top. Correction is a teaching strategy used to eliminate a mathematical error that has occurred (Peng & Luo, 2009). An error defined in class can be addressed either pragmatically and in a results-oriented way or analytically in a process-oriented way. In the second approach, the error is analyzed and its prevention is attempted, but it is not directly corrected (Rach et al., 2013). The role of error in the learning process in the classroom is a key point in preventing errors.

Matteucci et al. (2015) evaluated the research on how error is handled in the classroom in two parts. The first approach concerns social situations for errors; the second approach involves situations for using errors effectively in student learning. The latter is important because, as in the constructivist approach, errors promote cognitive conflict and are part of the learning process (Matteucci et al., 2015). It is clear that errors are effective for learning, but it is not clear how errors should be used in the classroom (Heemsoth & Heinze, 2016). In addition to using student mistakes, incorrect examples can also be used during teaching (Heemsoth & Heinze, 2014). Error-based activities also use incorrect examples after the correct examples are given. This practice is effective if students fully understand the concepts and study the correct examples (Große & Renkl, 2007).

The essence of EBA, which provides expertise in negative knowledge, is learning from mistakes. Examples with incorrect solutions are given to the student, and then the student is asked to find the errors made and give the correct solution. Presenting correct and incorrect examples together ensures that knowledge is learned in a consistent structure. The most critical point here is that an error can be structured so as to prevent further errors during the learning process and can be used in the lesson. The student can be enabled to recognize what is wrong, avoid these actions, and turn to what is right (Akpınar & Akdoğan, 2010; Heinze & Reiss, 2007; Yıldırım, 2019). Thus, the concept of error, the existence of which may have previously caused negative thinking, can now be considered as conveying a positive message that correct learning can be achieved (Doğan-Fırat, 2011).

EBA emerges as teaching practices that structure knowledge. To ensure this structuring, correct and incorrect information is presented together and both types of knowledge are learned. In this way, errors made are transformed into beneficial structures (Dalehefte et al., 2012; Gedik, 2014). This approach aims to ensure that individuals create accurate knowledge of themselves. Errors are thus an aid to correct information in creating knowledge. The point to be considered here is not to avoid making mistakes; it is to be aware of the mistake made (Akpınar & Akdoğan, 2010; Gedik, 2014; Özkaya et al., 2022). Preventing errors is contrary to knowledge structuring, which is the main principle of the constructivist approach (Heinze, 2005). For this reason, the idea that errors are to be prevented and avoided should not be imposed on the student. Otherwise, the student will not have the opportunity to explore and evaluate their misconceptions (Tulis, 2013).

When empirical research on error-based activities was examined, Durkin and Rittle-Johnson (2012) showed that students learned the concept of decimal representations and its key components better in a classroom in which errors were used in teaching. Heemsoth and Heinze (2014), who investigated fractions lessons in the seventh and eighth grades, experimentally showed that teaching with errors played a role in the acquisition of negative knowledge by students with sufficient prior knowledge. Again, it has been shown that students who adopt learning from their own errors are experimentally superior in terms of conceptual learning (Heemsoth & Heinze, 2016). In addition to these studies in which there are cognitive differences, there are also studies in which there is an affective difference instead of a cognitive difference. Rach et al. (2013) revealed that after the error-based activity (EBA) training they had provided teachers, the student fear of making errors decreased and the teacher behaviors were perceived as positive. In their quasi-experimental study demonstrating the effectiveness of error-based activities, Heinze and Reiss (2007) investigated the effectiveness of teacher-implemented error-based activities in middle and high school students. In this study, although there was no significant difference between the experimental and control groups in terms of cognition, there were differences in terms of affect. This situation shows that a positive error climate should be created in classrooms where error-based activities are implemented. Thus, the affective effects of error-based activities can also be examined.

Students and teachers must develop positive attitudes toward mistakes, that is, a positive error climate must be created in the classroom to adopt the approach of learning from errors (Guzmán-Muñoz et al., 2009; Tulis, 2013). With such a positive attitude, students do not hesitate to make errors. Correct perspectives can then be gained by analyzing the errors made and the necessary corrections can be made (Soylu & Soylu, 2006). The possible effects of the perceived error climate in the classroom, as revealed by Steuer et al. (2013), are shown in Figure 1.

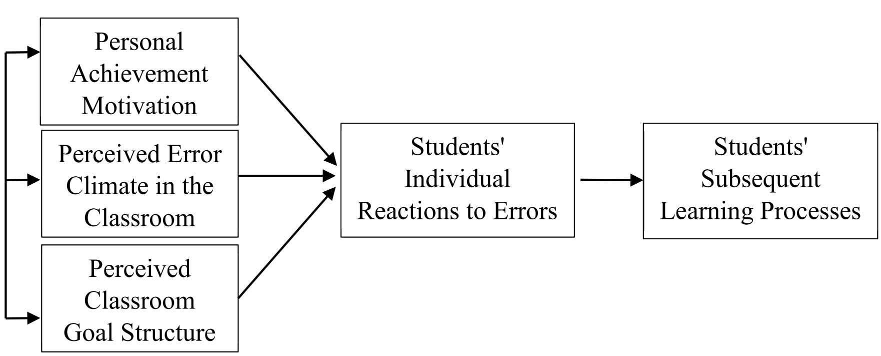


Figure 1
Expected Effects of Perceived Error Climate Source: Steuer et al. (2013)

It may be that error-based activities will positively change the perceived error climate in the classroom. It is important to reveal the affective results, as mentioned. It has been shown that students do not hesitate to make mistakes in a class with a positive error climate, especially in mathematics (Heinze & Reiss, 2007; Rach et al., 2013). Again, it was determined that students in a positive error climate developed a positive attitude toward mathematics (Özkaya et al., 2022) and had high motivation toward the course (Turner et al., 1998). Another affective situation that should be considered in a classroom where error-based activities are practiced is mathematics anxiety. Math anxiety is caused by the student, the teacher, or the teaching (Harris & Harris, 1987). Error-based activities are directly related to these three situations (Palkki & Hästö, 2019). It has been found that mathematics anxiety decreases in mathematics lessons in non-traditional classroom environments (Batton, 2010).

1.3 Research Questions

The main purpose of this study is to seek an answer to the question "What is the impact of EBA applications used in teaching fractions to sixth-grade students in terms of some variables?" In this context, answers to the following sub-research questions were sought.

1. What is the effect of EBA applications on sixth-grade student success in fractions?
2. What is the effect of EBA applications on the perceived error climate in the classroom?
3. What is the effect of EBA applications on sixth-grade student math anxiety levels?
4. What is the effect of EBA applications on the persistence of fraction understanding?
5. What are the opinions of sixth-grade students about EBA applications?

2 Methods

The quantitative aspect of the study was aimed to examine the effects of EBA applications on mathematics achievement, math anxiety levels, classroom error climate, and permanence of knowledge. For this purpose, a quasi-experimental design with a pretest-posttest control group was used. Certain variables that could be affected in these patterns were tested. Lessons were carried out with EBA applications in the experimental group to determine the effect of the independent variables. In the control group, the curriculum lessons were carried out and no other intervention was made. In this research, pre-tests were applied to the experimental and control groups before the

EBA application lessons were started, and post-tests were applied after the lessons were taught. The relevant data were collected.

The qualitative dimension of this study was a case study to determine student attitudes toward errors. After the post-tests were applied to the groups, semi-structured interviews were conducted with voluntarily selected students from the experimental group. Student opinions about EBA applications were thus obtained.

2.1 Participants

The "sequential mixed method sampling" method was used to determine the sample of this mixed method research. In this method, sample selection is made sequentially using probability and purposeful sampling methods. With this technique, a sample of the quantitative research becomes the determining factor for the subsequent qualitative research (Baki & Gökçek, 2012; Kemper et al., 2003). This research was aimed at selecting information-rich situations and examining them in depth (Büyüköztürk et al., 2020). The research group consisted of 44 sixth-grade students, 21 from the experimental group and 23 from the control group. In the qualitative dimension of the research, nine students from the experimental group were selected for interviews. While selecting these students, the changes between the pre-tests and post-tests results and the changes in their participation in the course were taken into consideration. Three groups were created: high, medium, and low level, and three students were randomly selected from each group. The selected students were asked whether they wanted to participate in the interview, and interviews were held following their voluntary consent. The students interviewed were categorized as L (low), M (medium), and H (high).

2.2 Data Collection Tools

Both quantitative and qualitative data collection tools were used in the research. The data collection tools of this study consisted of Subject Comprehension Test, Classroom Error Climate Student Questionnaire, Mathematics Anxiety Scale, and Semi-Structured Interview Questions.

2.2.1 Subject Comprehension Test (SCT)

The Subject Comprehension Test (SCT), consisting of open-ended questions, was prepared by the researcher and an expert in the field in order to determine the success of sixth-grade secondary school students in fractions. Using open-ended questions is more effective in measuring high-level behaviors such as analysis, synthesis, and evaluation (Atılgan et al., 2007). In particular, the phrase "Explain how you did it" was added to the end of the questions. The student answers were thus examined in depth. The last version of the SCT was evaluated by an expert in the field after the pilot application and was prepared for use for the main study. It was applied at pre-test and post-test. A scoring key for the SCT was prepared by the researchers. The scoring key was created to check the existence and accuracy of the solution and an explanation for each question. The scoring key was examined by an expert and finalized. In order to determine the reliability of the scoring by the researchers in line with the scoring key, 14 exam papers randomly selected from the pilot study were read by a mathematics teacher other than the researchers. The correlation coefficient of the scores made independently of each other was calculated. The determination of an agreement between scorers is presented as evidence of reliability (McMillan & Schumacher, 2010). The results showed a positive significant relationship between the two raters [$r = 0.924$, $p < 0.01$]. According to this result, the consistency between raters is quite high. Based on all the validity and reliability studies, it was decided that the SCT was appropriate for use in the main study. Scoring key is given in Table 1.

Table 1
The scoring key

	EXPLANATION			
	No Explanation (0)	False Explanation (1)	Partially True/ Incomplete Explanation (2)	Complete/ Accurate Explanation (3)
No Solution (0)	0	1	2	3
Incorrect Solution (1)	1	2	3	4
Incomplete/Partly Correct Solution (2)	2	3	4	5
Complete/Correct Solution (3)	3	4	5	6

2.2.2 Classroom Error Climate Student Questionnaire (CECSQ)

In order to determine the effect of EBA on the perceived error climate in the classroom, the Classroom Error Climate Student Survey (CECSQ), developed by Steuer et al. (2013) and adapted into Turkish by Kalaç et al. (2022), was used. Validity and reliability studies of the scale were conducted by Kalaç et al. (2022), and Cronbach's α value was calculated as 0.86. In order to recalculate the reliability of the scale, the Cronbach's α value was calculated by the researcher as 0.92 with data obtained from 43 students who were not in the experimental or control groups. According to Field (2009), if the Cronbach α coefficient calculated for a test is equal to or greater than 0.70, the reliability of the scale is at an acceptable level. Therefore, it was decided to use the scale in the main study.

2.2.3 Mathematics Anxiety Scale (MAS)

In order to determine the effect of EBA on the level of anxiety toward mathematics, the Mathematics Anxiety Scale (MAS) created by Bindak (2005) was used. The validity and reliability studies of the scale were tested by Bindak (2005), and Cronbach's α value was calculated as .84. Cronbach's α value was recalculated by the researcher with the data obtained from 43 students who were not in the experimental or control groups and was found to be .87. According to the Cronbach α coefficient, it can be concluded that the MAS is quite reliable for measuring the level of anxiety toward mathematics. Therefore, it was decided to use MAS in the main study.

2.2.4 Semi-Structured Interview Questions (SSIQ)

Semi-structured interviews were conducted to determine student thoughts about EBA applications. SSIQ were prepared by the researchers and an expert in order to collect the qualitative data of the study and describe the obtained quantitative data. After the post-tests were administered, interviews were held with nine volunteer students from the experimental group. The interviews were audio-recorded to ensure an objective process, and data loss prevention was attempted. It was stated that students would be given pseudonyms.

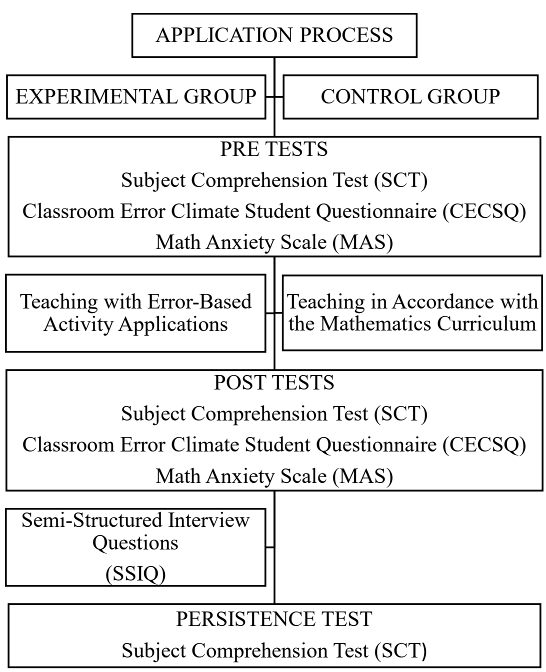
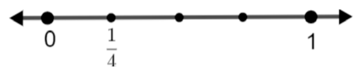


Figure 2
Process of Study

1. Through error-free question solutions prepared classroom activity

Question: Show the fraction $\frac{1}{4}$ on the number line.

Solution: First, we must check whether the given fraction is a simple or compound fraction is a simle or compound fraction. Since the numerator of the given fraction is less than the denominator, it is simple fraction. Simple fraction; It is means dividing the denominator of a whole into equal parts and considering the numerator. In other words, we need to divide the range range 0-1 on the number line into four equal parts and determine the first part.



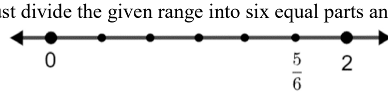
The range 0-1 on the number line into 4 equal parts and the first part is marked.

2. Prepared based on wrong question solutions classroom activity

Error Phase: The teacher ask the students to read the given question and think about the solution.

Question: Show the fraction $\frac{5}{6}$ on the given number line.

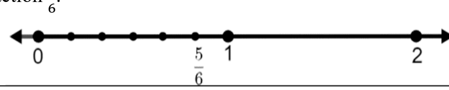
Solution: We must divide the given range into six equal parts and mark the fifth part.



Discussion Phase: The teacher asks the students to determine whether there is an error in solving this problem and, if so, what the reason. The teacher expects students to express their opinions through questioning, discussion and active learning methods.

Evaluation Phase: Based on the feedback from the students for the incorrect solution, it evaluates this solution with all students in the class.

Giving the Correct Answer Phase: The teacher presents the correct solution to the problem to the student. The given fraction is a simple fraction and lies between 0-1. The integer 1 should be written on the number line. Then, the area between 0 and 1 is divided into 6 equal parts and the fifth part is marked. In this way, we show the location of the fraction $\frac{5}{6}$.



Explaining the Reason for the Error Phase: The teacher explains the reason for the incorrect solution to the students. When showing the place of a fraction, a whole is taken and divided into equal parts equal to the number in the denominator. However, it was not noticed that the numbers on the number line given in the above solution do not consist of consecutive integers. Therefore, a correct solution could not be made. The range 0-1 should be divided into six equal parts and the fifth part should be marked.

Figure 3

Sample activity for the experimental group

2.3 Process

The study lasted five weeks. In the experimental group, incorrect and correct question solutions were given together and the students were asked to discuss these solutions and find the reasons for the error and the correct solution. Afterward, the correct solution was shown and the reasons for the errors made in the incorrect solution were explained. While the lessons were conducted with EBA applications in the experimental group, the lessons simply followed the mathematics curriculum in the control group: the correct solution of the given problem was made, but incorrect solutions were not given. The process of study is shown in Figure 2 and an example application in the experimental group is shown in Figure 3.

2.4 Analysis of Data

The analysis method to be applied to the quantitative data obtained from SCT, CECSQ, and MAS was chosen by looking at normality conditions. Qualitative data were subjected to content analysis.

2.4.1 Analysis of SCT

Considering that the difference scores exhibited a normal distribution, a dependent samples *t*-test was conducted in order to determine whether there was a significant difference between the pretest-posttest scores and the persistence test scores. An independent samples *t*-test was conducted to compare SCT scores in independent samples. The tests used within and between groups are shown in Figure 4.

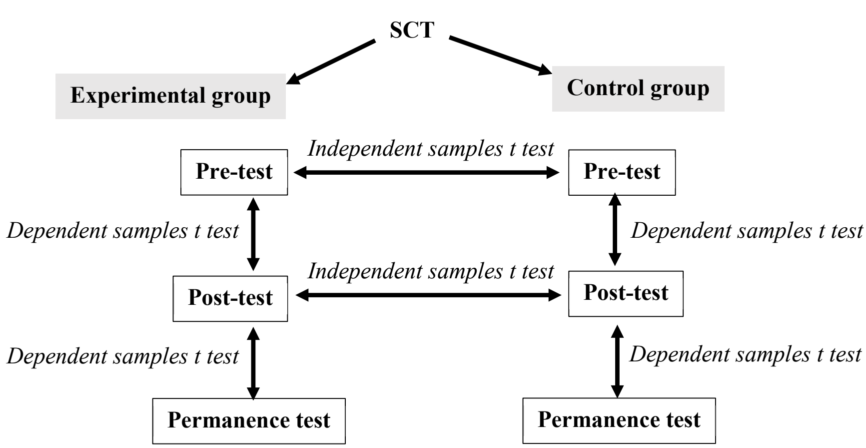


Figure 4
Tests used in the analysis of SCT

2.4.2 Analysis of CECSQ

Since the difference scores showed a normal distribution, a dependent samples *t*-test was conducted to determine whether the difference between the pre-test and post-test scores in dependent samples was significant. In addition, since the post-test scores of the experimental group did not exhibit a normal distribution, the Mann-Whitney U-test used to determine whether there was a significant difference between the post-test CECSQ scores of the experimental and control groups. An independent samples *t*-test was conducted to compare the pre-test CECSQ scores of the groups. The tests used within and between groups are shown in Figure 5.

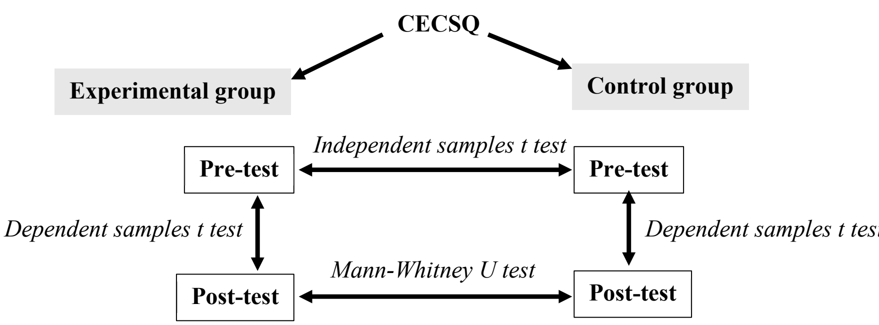


Figure 5
Tests used in the analysis of CECSQ

2.4.3 Analysis of MAS

Since the pre and post-test scores of the control and experimental groups did not show a normal distribution, the difference between the pre and post-test scores of both groups was evaluated using the Mann-Whitney U test. The pre-test and post-test difference scores in the control group showed normal distribution, but not in the experimental group. The tests used within and between groups are shown in Figure 6.

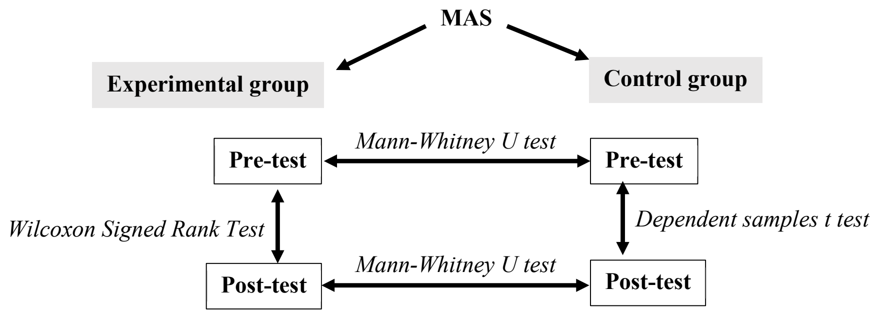


Figure 6
Tests used in the analysis of MAS

2.4.4 Analysis of semi-structured interviews

Content analysis is defined as summarizing and stating the basic content of the written data and the messages it contains (Cohen et al., 2005). With content analysis, the available data can be coded and managed (Büyüköztürk et al., 2020). The resulting codes can be organized, defined, and interpreted under certain themes (Yıldırım & Şimşek, 2011). In this research, the audio recordings obtained from the interviews were converted into written text documents by the researcher and the answers were transferred to a computer environment in direct quotes. Coding was done to analyze the answers given. Student statements were frequently included in the findings section. The data obtained from the interviews were analyzed by an expert in the field and reliable results were attained.

3 Findings

3.1 Findings regarding the effect of EBA applications on student achievement

In this section, an answer was sought to the research problem "What is the effect of EBA applications on the achievement of sixth-grade students in fractions?"

Table 2
SCT Independent t-test

Test	Group	*N*	*M*	*SD*	*df*	*t*	*p*
Pre-Test	Experimental	21	25.333	10.827	42	−0.099	0.922
	Control	23	25.652	10.598			
Post-Test	Experimental	21	73.762	25.420	42	2.582	0.013
	Control	23	56.522	18.628			

As the SCT was applied as a pre-test, no statistically significant difference was observed between the pre-test SCT average of the experimental group $M = 25.33$ and the pre-test SCT average of the control group ($M = 25.65$) [$t_{(42)} = -0.099$, $p > 0.05$]. In other words, the groups were equal in terms of success before the study started. When the SCT was applied as a post-test, a statistically significant difference was observed between the post-test SCT average score of the experimental group ($M = 73.76$) and the post-test SCT average score of the control group ($M = 56.52$) in favor of the experimental group [$t_{(42)} = 2.582$, $p < 0.05$] (see Table 2). However, the effect size was calculated as η^2=.137, which shows that the difference between the post-test scores of the two groups was moderate.

Table 3

Pre-test and post-test SCT-dependent t-test

Group	Test	*N*	*M*	*SD*	*df*	*t*	*p*
Experimental	Pre-Test	21	25.333	10.827	20	−13.183	0.000
	Post-Test	21	78.762	25.420			
Control	Pre-Test	23	25.652	10.598	22	−9.034	0.000
	Post-Test	23	56.522	18.628			

There was a significant difference between the mean of pre-test scores ($M = 25.33$) and the mean of post-test scores ($M = 78.76$) in the experimental group [$t_{(20)} = -13.183, p < 0.05$] (see Table 3). The effect size of this difference was calculated as $\eta^2 = 0.897$. Almost 90% of the significant difference between the pre-test and post-test scores is probably due to the teaching method (teaching with EBA applications). In the control group, there was also a significant difference between the mean of pre-test scores ($M = 25.65$) and the mean of post-test scores ($M = 56.52$) in favor of the post-test [$t_{(22)} = -9.034$, $p < 0.05$] (see Table 3). The effect size of this difference was calculated as $\eta^2 = 0.788$. Almost 79% of the significant difference between the pre-test and post-test scores is probably due to the teaching method (teaching by the curriculum). Considering that the calculated eta square effect sizes are 0.897 in the experimental group and 0.788 in the control group, it can be concluded that teaching with EBA applications is more effective for learning.

3.2 Findings regarding the effect of EBA applications on the perceived error climate in the classroom

In this section, an answer was sought to the research problem "What is the effect of EBA applications on the error climate perceived in the classroom?"

Table 4
Pre-test CECSQ independent t-test

Groups	*N*	*M*	*SD*	*df*	*t*	*p*
Experimental	21	104.641	19.576	42	0.868	0.390
Control	23	99.904	16.615			

As the CECSQ was applied as a pre-test, there was no statistically significant difference between the pre-test CECSQ average of the experimental group ($M = 104.64$) and the pre-test CECSQ average of the control group ($M = 99.90$) [$t_{(42)} = 0.868, p > 0.05$] (see Table 4). In other words, the groups were equivalent to each other in terms of the perceived error climate in the classroom before the study started.

Table 5
Post-test CECSQ Mann-Whitney U Test

Groups	*N*	Rank Average	Rank Sum	*U*	*z*	*p*
Experimental	21	29.57	621	93	−3.491	0.000
Control	23	16.04	369			

There was a statistically significant difference between the scores of the experimental group (Median: 123) and the control group (Median: 94) ($U = 93$, $p < 0.05$) (see Table 5). The effect size of this difference was found to be $r = 0.53$. The calculated r value shows that the difference between the post-test scores of these two groups is a high-level difference in favor of the experimental group. When compared to the control group in terms of classroom error climate, it can be said that a positive attitude toward errors was developed in the experimental group.

Table 6
CECSQ dependent t-test

Groups	Test	*N*	*M*	*SD*	*df*	*t*	*p*
Experimental	Pre-Test	21	104.641	19.576	20	−8.620	0.000
	Post-Test	21	116.454	17.839			
Control	Pre-Test	23	99.904	16.615	22	1.385	0.180
	Post-Test	23	93.518	18.439			

The dependent samples *t*-test of the CECSQ showed a significant difference between the mean of the pre-test ($M = 104.64$) and the mean of the post-test ($M = 116.45$) in favor of the post-test in the experimental group [$t_{(20)} = -8.620$, $p < 0.05$] (see Table 6). The effect size of this difference was calculated as

η^2=.788. There was no significant difference between the mean of the pre-test (M = 99.90) and the mean of the post-test (M = 93.52) in the control group of CECSQ [$t_{(22)}$ = 1.385, p > 0.05] (see Table 6). Considering that while no difference was observed in the control group, the eta squared effect size calculated in the experimental group was 0.788 and that this effect created a significant difference in favor of the post-test, it can be concluded that teaching with EBA applications is effective in the formation of a positive error climate in the classroom.

3.3 Findings regarding the effect of EBA applications on math anxiety

In this section, an answer was sought to the research problem "What is the effect of EBA applications on sixth-grade student math anxiety levels?"

Table 7

MAS Mann-Whitney U test

Test	Groups	*N*	Rank Average	Rank Sum	*U*	*z*	*p*
Pre-Test	Experimental	21	19.29	405	174	−1.588	0.112
	Control	23	25.43	585			
Post-Test	Experimental	21	14.90	313	82	−3.759	0.000
	Control	23	29.43	677			

The Mann-Whitney U test performed for the scores obtained from the pre-test MAS showed no statistically significant difference between the scores of the experimental group (Median: 18) and the control group (Median: 29) (U = 174, p > 0.05). The experimental and control groups were equal in terms of math anxiety levels before starting the study (Table 7).

A statistically significant difference was observed between the scores of the experimental group (Median: 13) and the control group (Median: 29) from the post-test MAS (U = 82, p < 0.05) (see Table 7). The effect size of this difference was calculated as r = 0.57. The calculated r value shows that the difference between the post-test scores is a high-level difference in favor of the control group. Therefore, there was a decrease in the math anxiety levels of the experimental group students as compared to the control group.

Table 8

MAS Wilcoxon signed-rank test applied to the experimental group

Pre-Test/Post-Test	*N*	Rank Average	Rank Sum	*z*	*p*
Negative Rows	21	11	231	−4.021	0.000
Positive Rows	0	0.0	0.0		
No Difference	0				

Table 9
MAS dependent t-test applied to the control group

Test	*N*	*M*	*SD*	*df*	*t*	*p*
Pre-Test	23	28.030	10.010	22	1.545	0.137
Post-Test	23	26.936	10.515			

There was a statistically significant difference between the pre-test and post-test scores of the experimental group participating in the study [$z = -4.021$, $p < 0.05$]. Considering the rank average and total of the difference scores, the fact that this difference is in negative ranks, that is, in favor of the pre-test, shows that EBA has a significant effect on reducing math anxiety. The effect size value is $r = 0.88$; the effect of the difference is large and 88% of the total variance is explained by the independent variable, EBA applications. In the control group, there was no significant difference in terms of math anxiety [$t_{(23)} = 1.545$, $p > 0.05$] (see Tables 8 and 9).

3.4 Findings regarding the effect of EBA applications on the permanence of knowledge
In this section, an answer was sought to the research problem "What is the effect of EBA applications on the permanence of fraction understanding?"

Table 10
Dependent t-test for permanence test and SCT post-test

Groups	Test	*N*	*M*	*SD*	*df*	*t*	*p*
Experimental	Post-Test	21	73.762	25.420	20	1.615	0.122
	Permanence Test	21	69.714	29.784			
Control	Post-Test	23	56.522	18.628	22	2.681	0.014
	Permanence Test	23	47.435	17.604			

According to the dependent sample *t*-test result of SCT, there was no significant difference between the average of post-test scores ($M = 73.76$) and the average of retention test scores ($M = 69.71$) in the experimental group [$t_{(20)} = 1.615$, $p > 0.05$]. In the control group of SCT, there was a significant difference between the mean of the post-test scores ($M = 47.44$) and the mean of the retention test scores () in favor of the post-test [$t_{(22)} = 2.681$, $p < 0.05$]. The effect size of this difference was calculated as η^2=.246. In this case, it can be concluded that teaching with EBA applications is more effective in the retention of knowledge.

3.5 Findings regarding the semi-structured interview process

In this section an answer was sought to the research problem "What are the opinions of sixth-grade students about EBA applications?" In order to find an answer, the categories, codes, students, and sample expressions resulting from the content analysis of the semi-structured interviews conducted with the experimental group after the post-tests were examined. Four categories were obtained: "cognitive effects of EBA," "affective effects of EBA," "effects of EBA on the quality of teaching services," and "field of application of EBA." Codes and individuals related to the category of cognitive effects of EBA are given in Figure 7.

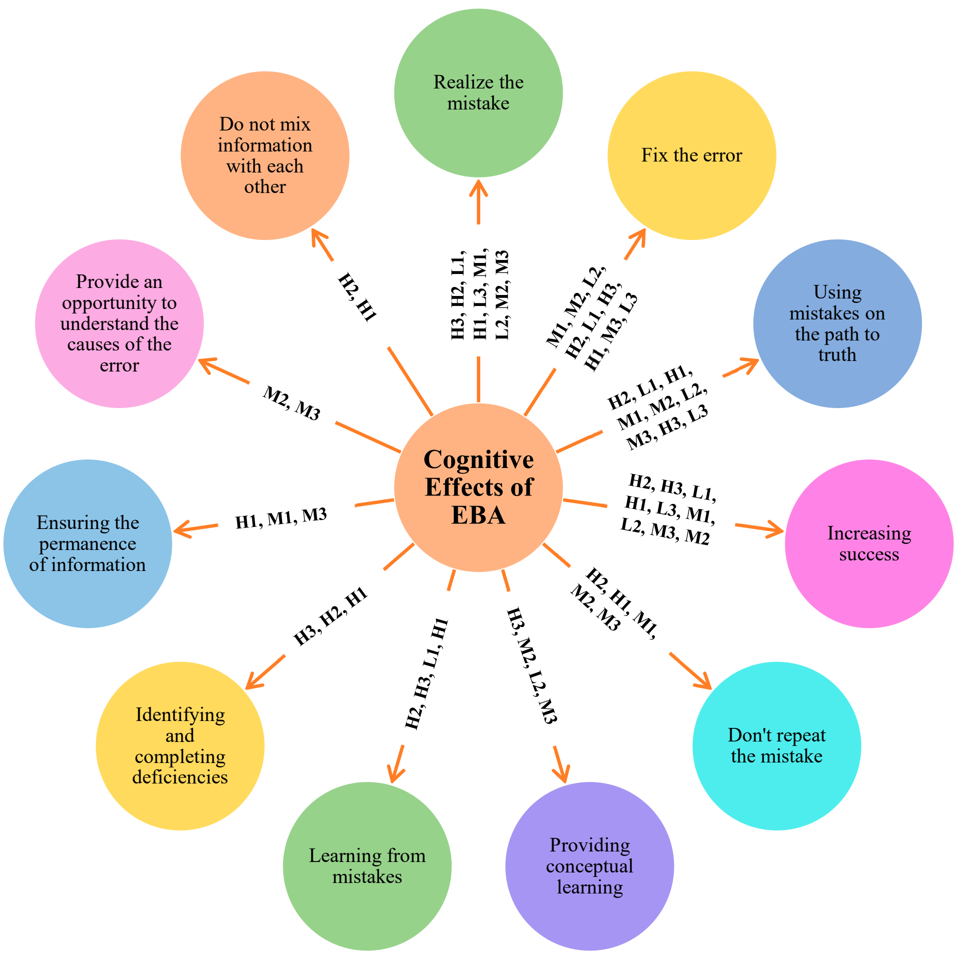


Figure 7

Codes belonging to the cognitive effects of EBA

Eleven codes emerged belonging to the cognitive effects of EBA category. Some student expressions belonging to this category are presented in Table 11.

Table 11
Codes and expressions related to the "Cognitive Effects of EBA" category

Codes	**Sample Expressions**
Realize the mistake	H1: *"I started to see my mistakes."*
Fix the error	L3: *"I started correcting our mistakes after you pointed them out."*
Using mistakes on the path to truth	H2: *"We learn a lot of information thanks to mistakes."*
Increasing success	M1: *"I think it was effective. At first, I added fractions without equalizing the denominators. I think mistakes help us learn."*
Don't repeat the mistake	M2: *"I am not making that mistake again."*
Providing conceptual learning	L2: *"When we make a mistake, we understand the basis of that problem."*
Learning from mistakes	H3: *"When we make a mistake, instead of being sad, we become happy because we learn from our mistakes."*
Identifying and completing deficiencies	H3: *"Since I am now doing right the things, I did wrong. I have completed my shortcomings."*
Ensuring the permanence of knowledge	M3: *"If we make a mistake, I learn the thing and never make that mistake again."*
Provide an opportunity to understand the causes of the error	M2: *"We need to know how we did right or wrong."*
Do not mix knowledge with each other	H2: *"I used to always mix fractions, but now I never mix them."*

Figure 7 shows that all the interviewed students gave answers in the codes of "realizing the mistake," "correcting the mistake," "using mistakes on the path to the truth," and "increasing success." The smallest number of students is included in the codes "providing an opportunity to understand the reasons for the error" and "not mixing information with each other." The success level of the two students in the code "providing an opportunity to understand the reasons for the error" is at a medium level. In the code "not mixing information with each other," there are two students with high success levels. Codes and students related to the affective effects of EBA category are given in Figure 8.

Figure 8

Codes and students for the "Affective Effects of EBA" category

Thirteen codes emerged belonging to the affective effects of EBA category. Some student expressions from this category are presented in Table 12.

Table 12
Codes and expressions related to the "Affective Effects of EBA" category

Codes	**Sample Expressions**
Reduce fear	H1: *"I used to be afraid of making mistakes and rarely got up to the board, but now I am not afraid at all."*
Increasing interest/desire	L2: *"I attended classes more willingly."*
Increasing self-confidence	L3: *"I even teach it to my brother. When people come to us, they always come to me and ask me questions, and I help them."*
Increasing motivation	L2: *"It increased my motivation even more."*
Looking forward to math lessons	L1: *"I always wanted to go into mathematics, mathematics is very good."*
Developing a positive attitude toward mistakes	H3: *"When we make a mistake, instead of being sad, we become happy because we learn from our mistakes."*
Entertain in class	M1: *"I think math lessons are more fun."*
Thinking the subject is easy	H1: *"I understood how easy it is to divide, multiply, add, subtract and order fractions."*
Make you feel happy	H3: *"I am happier now because I have no shortcomings."*
Increased belief in ability to do	M1: *"Even if I make a mistake, I think I will find the truth immediately."*
Don't hesitate	M3: *"In the past, when a question appeared on the board, I would hesitate a little more, wondering if I would make a mistake. But now, even if a make a mistake, I get up knowingly to learn the right thing."*
Reduce anxiety	H2: *"I used to be more anxious, now I have no more anxiety."*
Reduce stress	H3: *"Teacher, I used to get a little stressed when I made mistakes, but when we learn to learn from our mistakes, nothing happens anymore."*

All of the interviewed students gave answers coded as "reducing fear" and "increasing interest/desire." The smallest number of students was in the "reducing stress" code, and the student with the highest level of success answered for this code.

The codes and students related to the category of EBA's effects on the quality of teaching services are presented in Figure 9.

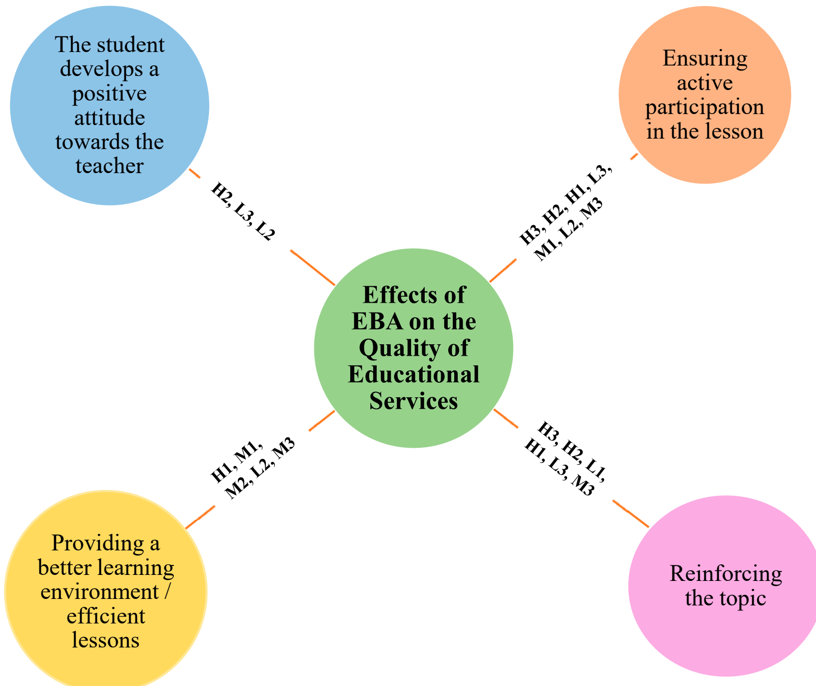


Figure 9

Codes and persons related to the category "The effects of EBA on the quality of teaching services".

Four codes emerged in the category of effects of EBA on the quality of teaching services. Some student expressions from this category are presented in Table 13.

Table 13

Codes and expressions related to the category "The effects of EBA on the quality of teaching services"

Codes	**Sample Expressions**
Ensuring active participation in the lesson	L2: *"I used to lift my fingers less, but now I lift them more."*
Reinforcing the topic	H1: *"We can see our mistakes and understand which ones we need to reinforce more."*
Providing a better learning environment/efficient lesson	M2: *"Mistakes help us learn the truth. I learn better. Before, we continued to make our mistakes and made them constantly."*
The student develops a positive attitude toward the teacher	D3: *"Teacher, for example, when we stand up at the board and make a mistake, you say, 'You made a very good mistake.'"*

Most of the interviewed student answers are in the code of "active participation in the lesson" and the success level of three of these students is high, two are medium, and two are low. The smallest number of students is within the code of "student developing a positive attitude toward the teacher" and one of these students has a high level of success and two of them have a low level of success.

Codes and people related to EBA's application field category are given in Figure 10.

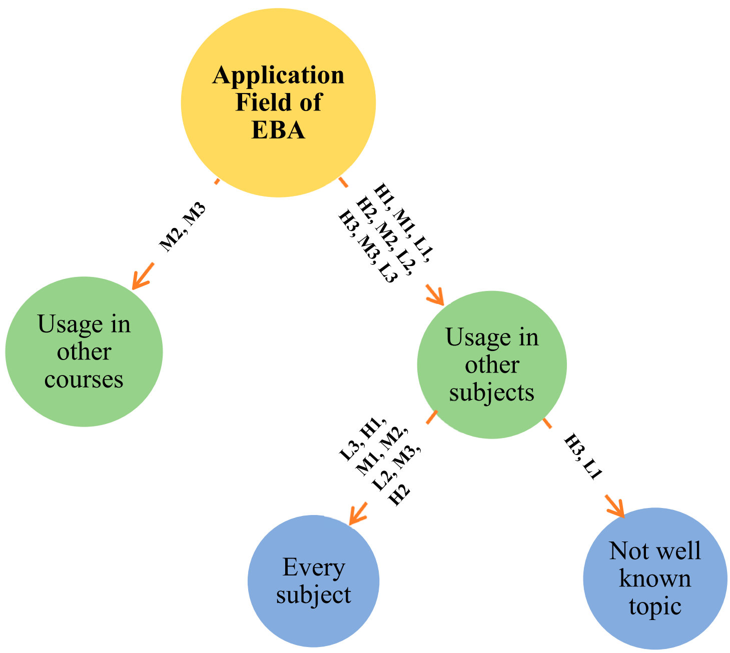


Figure 10
Codes and persons related to the "Application field of EBA" category.

Two codes emerged belonging to the application field category of EBA. Some student expressions from this category are presented in Table 14.

Table 14
Codes and expressions related to the "Application field of EBA" category

<table>
<tr><th>Codes</th><th colspan="2">Sample Expressions</th></tr>
<tr><td>Usage in other courses</td><td colspan="2">M3: "I would like it to happen in other lessons as well because in other lessons we see our mistakes and we turn to the right way and do not make that mistake again."</td></tr>
<tr><td rowspan="2">Usage in other subjects</td><td rowspan="2">L3: "We need to learn through mistakes, not only on fractions but also on different subjects."</td><td>(Every subject)
M1: "Teacher, I think the faulty solutions used in the course are necessary. Mistakes help us learn, whether it's a subject we know or a subject we don't know."</td></tr>
<tr><td>(Not well-known topic)
H3: "If it's something we don't know, we need to see it to learn. If our knowledge about that subject is good, it may be possible even if we do not see it."</td></tr>
</table>

Nine students are in the "usage in other subjects" code. Two students used expressions included in the code "usage in other courses" and the success level of both of these students was medium. In the "usage in other subjects" code, seven students used expressions in the "every subject" code and two students use expressions in the "well-known subject" code. The achievement level of the students in the "every subject" code is high for two, medium for three, and low for two. The success level of the students in the "not well-known subject" code is high for one and low for the other.

4 Discussion and Conclusions

This study aimed to examine the effects of EBA applications in teaching fractions in sixth grade on student success, in-class error climate, math anxiety, and retention of knowledge, and to determine student opinions about EBA applications. For this purpose, in order to find answers to the research questions, data were collected using SCT, CECSQ, MAS, and SSIQ data collection tools and findings were presented. In this part of the research, the findings are discussed comprehensively and a conclusion is reached.

Based on the results of SCT and the interviews, teaching with EBA applications is effective in increasing the success levels of students. When looking at the literature, different studies support this result. Akkuşci (2019) asserted that EBA applications have a positive effect on success. In another study, Yıldırım (2019) concluded that the incorrect solution method contributes to better learning of the subject. In another study, Altıntaş et al. (2020) stated that there is a positive relationship between academic success and the scores obtained from the EBA test. Similarly, a study conducted by Karadağ (2004) with seventh grade students found that the group in which mistakes were explained and discussed was more successful, and it was stated that mistakes were learning opportunities. McLaren et al. (2015) stated that EBA had a positive effect on the experimental group and that the experimental group performed better than the control group. Similarly, Durkin and Rittle-Johnson (2012) concluded that giving incorrect examples as well as correct examples is more effective in learning. Apart from this, Heinze and Reiss (2007) determined that the group trained with EBA had better reasoning and proof skills than the control group and that the students were more successful in complex mathematical problems. Borasi (1989) examined the structuring and use of errors in his study and found that students became aware of mistakes, gained new ideas, and developed questioning skills; it was stated that mistakes can help success. Akpınar and Akdoğan (2010) mentioned that using mistakes can provide new expansions that can increase low success. Özkaya and Konyalıoğlu (2019) stated that thanks to EBA, teachers were able to realize their own conceptual levels. EBA is thus reportedly effective in teaching a new concept if there is a certain level of readiness. Gürbüz et al. (2021) concluded that EBA applications are more effective in learning than traditional teaching method. In their study, Steuer et al. (2013) concluded that a positive error climate in the classroom had a positive effect on students' learning. The study conducted by Francome and Hewitt (2020) showed that students valued mistakes, and it was concluded that they developed a more positive attitude toward mistakes, that mistakes help them, and that they see mistakes as opportunities for learning, meaning that better learning can occur. Gedik et al. (2017) also mentioned that EBA has a positive effect on students both cognitively and affectively. The results of this study are therefore consistent with these listed studies (Akkuşci, 2019; Akpınar & Akdoğan, 2010; Altıntaş et al., 2020; Borasi, 1989; Durkin & Rittle-Johnson, 2012; Francome & Hewitt, 2020; Gedik et al., 2017; Gürbüz et al., 2021; Heinze & Reiss, 2007; Karadağ, 2004; McLaren et al., 2015; Özkaya & Konyalıoğlu, 2019; Steuer et al., 2013; Yıldırım, 2019) and show that EBA applications have positive effects on learning and student achievement.

Based on the results of CECSQ and the interviews, EBA applications positively affect the error climate in the classroom. Other studies support this result. Durkin and Rittle-Johnson (2012) observed that students were more active in a suitable discussion environment when incorrect examples were given as well as correct examples. Rach et al. (2013) considered mistakes as learning opportunities. The research showed that the error-tolerant classroom culture increased to effective levels and this classroom environment had a positive effect on the student's affective level. In the study conducted by Steuer et al. (2013), the perceived error climate in the classroom affected students in terms of effort and self-regulation. The study by Gedik et al. (2017) showed that EBA has a positive effect on students both cognitively and affectively, and it was concluded that EBA has a motivating effect on students' mathematics learning and provides them with an innovative, interesting, and critical perspective. Heinze (2005) concluded that low-achieving students are not aware that mistakes are an opportunity to learn. The study conducted by Francome and Hewitt (2020) concluded that students value mistakes, develop positive attitudes toward mistakes, can be helped by mistakes, and see mistakes as opportunities for learning. The study by Heinze and Reiss (2007) found that teacher attitudes toward errors can have a positive impact on student success. Our results were consistent with those of these listed studies (Durkin & Rittle-Johnson, 2012; Francome & Hewitt, 2020; Gedik et al., 2017; Heinze, 2005; Heinze & Reiss, 2007; Rach et al., 2013; Steuer, et al., 2013), showing that EBA applications have a positive effect on the formation of a positive error climate in the classroom.

Based on the MAS and interviews, teaching with EBA applications is effective in reducing student math anxiety. Studies have showed that the use of errors affects students positively in terms of affective aspects. Gedik (2014) found that EBA practices had a positive impact on secondary school teachers in terms of cognitive and affective aspects and had a motivating effect. Akkuşci (2019) concluded that EBA positively affects students cognitively and affectively. The study by Rach et al. (2013) observed that the fear of making mistakes decreased in the experimental group of students with a positive error climate. In another study, Gedik et al. (2017) found that EBA had a positive effect on students both cognitively and affectively. Similarly, Heinze (2005) stated that if teachers do not allow mistakes students are afraid of making mistakes. Based on this result, students will not be afraid if making mistakes is not prohibited. Durkin and Rittle-Johnson (2012) observed that students were more active in a suitable discussion environment when incorrect examples were given. This result is consistent with this study, which concluded that students overcame the anxiety of making mistakes and therefore participated more in the lesson. The studies mentioned above (Akkuşci, 2019; Durkin & Rittle-Johnson, 2012; Gedik, 2014; Gedik et al., 2017; Heinze, 2005; Rach et al., 2013) showed that EBA applications affect students positively. It affects people's anxiety, decreasing their anxiety levels.

The permanence test SCT and interviews indicated that teaching with EBA applications has a positive effect on the permanence of knowledge. McLaren et al. (2015) found that EBA had a positive effect on the experimental group and provided more opportunities for knowledge to remain in long-term memory than the traditional teaching method. Gedik (2014) stated that EBA is effective in making learning more permanent. The results of Heinze's (2005) study supported the results of this study regarding the permanence of knowledge. Borasi (1989) observed that by structuring and using errors, knowledge becomes more permanent and there is an increase in the level of curiosity and interest in knowledge. The results of this study were found to be compatible with the studies mentioned above (Borasi, 1989; Gedik, 2014; Heinze, 2005; McLaren et al., 2015). Therefore, EBA has a positive effect on the permanence of knowledge. EBA positively affects students cognitively and affectively, increases the quality of teaching services, and can be used in different subjects and different mathematics lessons, according to the results of the semi-structured interviews.

Note

This study was produced from the first author's master's thesis. A part of this study was presented as a summary paper at TURCOMAT-6.

References

Abay, J. R., & Clores, M. A. (2022). Beliefs, attitudes, and practices of high school teachers in handling students' errors: Implications for error-tolerant mathematics classrooms. *International Journal on Emerging Mathematics Education, 6*(2), 101–118. http://dx.doi.org/10.12928/ijeme.v6i2.23995

Akkuşci, Y. E. (2019). *Matematik öğretiminde hata temelli aktivite uygulamalarının sınıf içi kullanımının etkililiğinin incelenmesi* [Yüksek lisans tezi, Atatürk Üniversitesi-Erzurum]. Yükseköğretim Kurulu Ulusal Tez Merkezi. https://tez.yok.gov.tr/UlusalTezMerkezi/tezDetay.jsp?id=4RrXVPh5V-RTWEb_yZSodg&no=qorqZAgEazaLB8KpuuxzKw

Akpınar, B., & Akdoğan, S. (2010). Negatif bilgi kavramı: Hata ve başarısızlıklardan öğrenme. *Batı Anadolu Eğitim Bilimleri Dergisi, 1*(1), 14–22.

Altıntaş, E., İlgün, Ş., & Daştan, Z. (2020). 7. Sınıf öğrencilerinin matematikte hatalı çözüm içeren sorularla imtihanı. *Turkish Studies-Educational Sciences, 15*(6), 3939–3957. https://doi.org/10.47423/TurkishStudies.46584

Atılgan, H., Kan, A., & Doğan, N. (2007). *Eğitimde ölçme ve değerlendirme* (2. baskı). Anı.

Baki, A., & Gökçek, T. (2012). Karma yöntem araştırmalarına genel bir bakış. *Elektronik Sosyal Bilimler Dergisi, 11*(42), 1–21.

Batton, M. (2010). *The effect of cooperative groups on math anxiety* [Doctoral dissertation, Walden University Minneapolis]. ERIC. https://eric.ed.gov/?id=ED523972

Bindak, R. (2005). İlköğretim öğrencileri için matematik kaygı ölçeği. *Fırat Üniversitesi Fen ve Mühendislik Bilimleri Dergisi, 17*(2), 442–448.

Borasi, R. (1987). Exploring mathematics through the analysis of errors. *For the Learning of Mathematics, 7*(3), 2–8.

Borasi, R. (1988, April). *Towards a reconceptualization of the role of errors in education: The need for new metaphors* [Conference session]. Annual meeting of the American educational research association. https://eric.ed.gov/?id=ED295969

Borasi, R. (1989, March). *Students' constructive uses of mathematical errors: A taxonomy* [Conference session]. Annual meeting of the American educational research association. https://eric.ed.gov/?id=ED309069

Borasi, R. (1994). Capitalizing on errors as "springboards for inquiry": A teaching experiment. *Journal for Research in Mathematics Education, 25*(2), 166–208. https://doi.org/10.2307/749507

Borasi, R. (1996). *Reconceiving mathematics instruction: A focus on errors*. Ablex Publishing Corporation.

Bray, W., & Santagata, R. (2013, January 24-26). *Developing teaching capacity for making productive use of mathematical errors* [Conference session]. Association of mathematics teacher educators annual meeting, Orlando Florida, USA.

Büyüköztürk, Ş., Kılıç-Çakmak, E., Akgün, Ö. E., Karadeniz, Ş., & Demirel, F. (2020). *Eğitimde bilimsel araştırma yöntemleri* (28. baskı). Pegem Akademi.

Cohen, L., Manion, L., & Morrison, K. (2005). *Research methods in education* (5th Ed.). Routledge. http://dx.doi.org/10.4324/9780203224342

Dalehefte, I. M., Seidel, T., & Prenzel, M. (2012). Reflecting on learning from errors in school instruction: Findings and suggestions from a Swiss-German video study. In J. Bauer & C. Harteis (Eds.), *Human Fallibility. Professional and Practice-based Learning, 6*, (pp. 197–213). Springer. https://doi.org/10.1007/978-90-481-3941-5_12

Doğan-Fırat, S. (2011). *Matematik derslerindeki öğrenci hatalarına karşı öğretmen tutumları* [Yüksek Lisans Tezi, Adıyaman Üniversitesi-Adıyaman]. Yükseköğretim Kurulu Ulusal Tez Merkezi. https://tez.yok.gov.tr/UlusalTezMerkezi/tezDetay.jsp?id=44vOs7dz3eWwEgeU_SH6aA&no=QY0piDfJNBL5w3uEKKmj1Q

Dresel, M., Schober, B., Ziegler, A., Grassinger, R., & Steuer, G. (2013). Affektiv-motivational adaptive und handlungsadaptive Reaktionen auf Fehler im Lernprozess [Affective-motivational adaptive and action adaptive reactions on errors in learning processes]. *Zeitschrift für Pädagogische Psychologie, 27*(4), 255–271. https://doi.org/10.1024/1010-0652/a000111

Durkin, K., & Rittle-Johnson, B. (2012). The effectiveness of using incorrect examples to support learning about decimal magnitude. *Learning and Instruction, 22*(3), 206–214. https://doi.org/10.1016/j.learninstruc.2011.11.001

Field, A. (2009). *Discovering statistics using SPSS* (3rd Ed). Sage Publications.

Francome, T., & Hewitt, D. (2020). "My math lessons are all about learning from your mistakes": How mixed-attainment mathematics grouping affects the way students experience mathematics. *Educational Review, 72*(4), 475–494. https://doi.org/10.1080/00131911.2018.1513908

Gartmeier, M., Bauer, J., Gruber, H., & Heid, H. (2008). Negative knowledge: Understanding professional learning and expertise. *Vocations and Learning*, *1*, 87–103. https://link.springer.com/article/10.1007/s12186-008-9006-1

Gedik, S. D. (2014). *Matematik alan bilgisi geliştirme sürecine hata temelli aktivitelerin etkisi* [Yüksek lisans tezi, Atatürk Üniversitesi-Erzurum]. Yükseköğretim Kurulu Ulusal Tez Merkezi. https://tez.yok.gov.tr/UlusalTezMerkezi/tezDetay.jsp?id=gAXFoMF71KS18kH2LpDIjA&no=8KHDF1J_tc0v-HNo8mN-9g

Gedik, S. D., Konyalıoğlu, A. C., Tuncer, E. B., & Morkoyunlu, Z. (2017). Mistake handling activities in mathematics educatıon: Practice in class. *Journal of Education and Human Development, 6*(2), 86–95. https://doi.org/10.15640/jehd.v6n2a9

Grassinger, R., & Dresel, M. (2017). Who learns from errors on a class test? Antecedents and profiles of adaptive reactions to errors in a failure situation. *Learning and Individual Differences*, *53*, 61–68. https://doi.org/10.1016/j.lindif.2016.11.009

Große, C. S., & Renkl, A. (2007). Finding and fixing errors in worked examples: Can this foster learning outcomes? *Learning and Instruction, 17*(6), 612–634. https://doi.org/10.1016/j.learninstruc.2007.09.008

Gürbüz, R., Yıldırım, İ., & Doğan, M. F. (2021). Hata temelli aktivitelerin 7. sınıf öğrencilerinin bazı istatistik kavramlarındaki başarısı üzerindeki etkisi. *Bilgisayar ve Eğitim Araştırmaları Dergisi, 9*(18), 997–1021. https://doi.org/10.18009/jcer.976155

Guzmán-Muñoz, F. J., Gruber, H., Heid, H., Lorenzer, K., Bauer, J., Heinze, A., Tuerling, J. M., Nägele, C., Baumgartner, A., Anja, M., Seifried, J., Link, M., Gartmeier, M., & Wuttke, E. (2009, September 25–30). *Learning from errors* [Conference session]. ECER, Vienna, Austria. https://eera-ecer.de/ecer-programmes/conference/2/contribution/2488

Harris, A., & Harris, J. (1987). Reducing mathematics anxiety with computer assisted instruction. *Mathematics and Computer Education, 21*(1), 16–24.

Heemsoth, T., & Heinze, A. (2014). The impact of incorrect examples on learning fractions: A field experiment with 6th grade students. *Instructional Science*, *42*, 639–657. https://doi.org/10.1007/s11251-013-9302-5

Heemsoth, T., & Heinze, A. (2016). Secondary school students learning from reflections on the rationale behind self-made errors: A field experiment. *The Journal of Experimental Education*, *84*(1), 98–118. https://doi.org/10.1080/00220973.2014.963215

Heinze, A. (2005). Mistake-handling activities in the mathematics classroom. *International Group for the Psychology of Mathematics Education, 3*, 105–112.

Heinze, A., & Reiss, K. (2007, July). Mistake-handling activities in the mathematics classroom: Effects of an in-service teacher training on students' performance in geometry. In J.-H. Woo, H.-C. Lew, K.-S. Park, & D.-Y. Seo (Eds.), *Proceedings of the 31st conference of the international group for the psychology of mathematics education, 3*, (pp. 9–16). PME.

Hodgen, J., & Askew, M. (2010). Assessment for learning: What is all the fuss about? In I. Thopson (Ed.), *Issue in teaching numeracy in primary* schools (2nd Ed.) (pp. 133–145). Open University. https://books.google.com.tr/books?hl=tr&lr=lang_en%7Clang_tr&id=gCNFBgAAQBAJ&oi=fnd&pg=PA133&ots=Ylp58KfxRR&sig=gvfQQiJfyUhsOMDRLtebrYXteUY&redir_esc=y#v=onepage&q&f=false tr&id=gCNFBgAAQBAJ&oi=fnd&pg=PA133&ots=Ylp58KfxRR&sig=gvfQQiJfyUhsOMDRLtebrYXteUY&redir_esc=y#v=onepage&q&f=false

Ingram, J., Pitt, A., & Baldry, F. (2015). Handling errors as they arise in whole-class interactions. *Research in Mathematics Education, 17*(3), 183–197. https://doi.org/10.1080/14794802.2015.1098562.

Käfer, J., Kuger, S., Klieme, E., & Kunter, M. (2018). The significance of dealing with mistakes for student achievement and motivation: Results of doubly latent multilevel analyses. *European Journal of Psychology of Education*, *34*(4), 731–753. https://doi.org/10.1007/s10212-018-0408-7

Kalaç, S., Özkaya, M., & Konyalıoğlu, A. C. (2022). Sınıf hata iklimi ölçeğinin Türkçe'ye uyarlama çalışması. *Atatürk Üniversitesi Kazım Karabekir Eğitim Fakültesi Dergisi, 44*(1), 100–109. https://doi.org/10.54614/AUJKKEF.2022.11-22

Karadağ, Z. (2004). *Hatalardan ögrenme yöntemi: Koordinat düzlemi ve simetri örneği.* 4th International educational technology symposium, Sakarya.

Kemper, E., Stringfield. S., & Teddlie, C. (2003). Mixed methods sampling strategies in social science research. In A. Tashakkori & C. Teddlie (Eds.), *Handbook of mixed methods in social & behavioral research* (pp. 273–296). Sage. https://doi.org/10.4135/9781506335193

Konyalıoğlu, A. C., Aksu, Z., Şenel, E. Ö., & Tortumlu, N. (2010). *Matematik öğretmen adaylarının matematik soru çözümlerinde yapılan hataların nedenlerini sorgulama becerilerinin incelenmesi.* Uluslararası Öğretmen Yetiştirme Politikaları ve Sorunları Sempozyumu II. Hacettepe Üniversitesi, Ankara.

Konyalıoğlu, A.C., Özkaya M., & Gedik, S. D.(2019). *Öğretmen ve öğretmen adayları için matematik öğretiminde hata temelli aktivitler.* Ertual Akademi.

Kurdal, C. (2016). *Dinamik ve etkileşimli matematik öğrenme ortamlarında öğrencilerin kesirler ve oran orantı konusunda yaptığı hatalar ve çözüm önerileri* [Yüksek lisans tezi, Bayburt Üniversitesi-Bayburt]. Yükseköğretim Kurulu Ulusal Tez Merkezi. https://tez.yok.gov.tr/UlusalTezMerkezi/tezDetay.jsp?id=asaeFbGKZyrBgmgAP5ebrA&no=n8RujVc7pjqJQj6Jr-gK5w

Matteucci, M. C., Corazza, M. & Santagata, R. (2015). Learning from mistakes, or not. An analysis of teachers' error beliefs and mistake-handling strategies through questionnaire and video. *Progress in Education, 37*(3), 33–54.

McLaren, B. M., Adams, D. M., & Mayer, R. E. (2015). Delayed learning effects with erroneous examples: A study of learning decimals with a web-based tutor. *International Journal of Artificial Intelligence in Education, 25*(4), 520–542. https://doi.org/10.1007/s40593-015-0064-x

McMillan, J. H., & S. Moore. (2020). Better being wrong (sometimes): Classroom assessment that enhances student learning and motivation. *The Clearing House: A Journal of Educational Strategies, Issues and Ideas*, *93*(2), 85–92. https://doi.org/10.1080/00098655.2020.1721414

McMillan, J., & Schumacher, S. (2010). *Research in education: Evidence-based inquiry* (7th Ed.). Pearson.

Melis, E., Sander, A., & Tsovaltzi, D. (2010). How to support meta-cognitive skills for finding and correcting errors. *Cognitive and Metacognitive Educational Systems: Papers from the AAAI Fall Symposium*, *1*, 64–68.

Metcalfe, J. (2017). Learning from errors. *Annual Review of Psychology, 68*, 465–489. https://doi.org/10.1146/annurev-psych-010416-044022

National Council of Teachers of Mathematics. [NCTM]. (2000). *Principles and standard for school mathematics*. National Council of Teachers of Mathematics.

Oser, F., & Spychiger, M. (2005). *Lernen ist schmerzhaft: Zur Theorie des Negativen Wissens und zur Praxis der Fehlerkultur [Learning is painful. On the theory of negative knowledge and the culture of mistakes in practice]*. Beltz.

Özkaya, M. (2015). *Hata temelli aktivitelerin matematik öğretmenlerinin mesleki gelişimlerine etkisinin incelenmesi* [Doktora tezi, Atatürk Üniversitesi-Erzurum]. Yükseköğretim Kurulu Ulusal Tez Merkezi. https://tez.yok.gov.tr/UlusalTezMerkezi/tezDetay.jsp?id=AEFLQMfQDB91qjnuoCTAeg&no=YVUL74awBpoCUkVZtgyOlg

Özkaya, M., & Konyalıoğlu, A. C. (2019). Ortaokul matematik öğretmenlerinin konu alan bilgilerinin gelişiminde hata temelli aktiviteler: Kesirlerle toplama işlemi. *Bayburt Eğitim Fakültesi Dergisi, 14*(27), 23–52. https://doi.org/10.35675/befdergi.475076

Özkaya, M., Kalaç, S., & Konyalıoğlu, A. C. (2022). The effect of positive error climate on affective domains in mathematics teaching. *International Journal of Assessment Tools in Education*, *9*(Special Issue), 236–257. https://doi.org/10.21449/ijate.1121828

Palkki, R., & Hästö, P. (2019). Mathematics teachers' reasons to use (or not) intentional errors. *Teaching Mathematic And Computer Science, 16*(2), 263–282. https://doi.org/10.5485/TMCS.2018.0453

Parviainen, J., & Eriksson, M. (2006). Negative knowledge, expertise and organisations. *International Journal of Management Concepts and Philosophy, 2*(2), 140–153. https://doi.org/10.1504/IJMCP.2006.010265

Peng, A. (2010). Teacher knowledge of students' mathematical errors. *MAVI 16, The 16th conference on mathematical views*. https://urn.kb.se/resolve?urn=urn:nbn:se:umu:diva-35221

Peng, A., & Luo, Z. (2009). A framework for examining mathematics teacher knowledge as used in error analysis. *For the Learning of Mathematics, 29*(3), 22–25.

Rach, S., Ufer, S., & Heinze, A. (2013). Learning from errors: Effects of teachers' training on students' attitudes towards and their individual use of errors. *PNA, 8*(1), 21–30. https://doi.org/10.30827/pna.v8i1.6122

Seifried, J., & Wuttke, E. (2010). Student errors: How teachers diagnose them and how they respond to them. *Empirical Research in Vocational Education and Training, 2*(2), 147–162. https://doi.org/10.25656/01:8250

Siegler, R. S. (2002). Microgenetic studies of self-explanation. In N. Granott & J. Parziale (Eds.), *Microdevelopment. Transition processes in development and learning* (pp. 31–58). Cambridge University Press. https://doi.org/10.1017/CBO9780511489709.002

Soylu, Y., & Soylu, C. (2006). Matematik derslerinde başarıya giden yolda problem çözmenin rolü. *İnönü Üniversitesi Eğitim Fakültesi Dergisi, 7*(11), 97–111.

Spychiger, M., Kuster, R., & Oser, F. (2006). Dimensionen von fehlerkultur in der schule und ihre messung. Der schülerfragebogen zur fehlerkultur im unterricht für mittel- und oberstufe [Dimensions of error culture at school and its measurement]. *Revue Suisse Des Sciences Del'éducation*, *28*(1), 87–110. https://doi.org/10.25656/01:4140

Steuer, G., Rosentritt-Brunn, G., & Dresel, M. (2013). Dealing with errors in mathematics classrooms: Structure and relevance of perceived error climate. *Contemporary Educational Psychology, 38*(3), 196–210. https://doi.org/10.1016/j.cedpsych.2013.03.002

Swan, M. (2002). Dealing with misconceptions in mathematics. In P. Gates (Ed.), *Issues in mathematics teaching* (pp. 147–165). Routledge.

Tulis, M. (2013). Error management behavior in classrooms: Teachers' responses to student mistakes. *Teaching and Teacher Education, 33*, 56–68. https://doi.org/10.1016/j.tate.2013.02.003

Tulis, M., Steuer, G., & Dresel, M. (2016). Learning from errors: A model of individual processes. *Frontline Learning Research*, *4*(2), 12–26. https://doi.org/10.14786/flr.v4i2.168

Turner, J. C., Meyer, D. K., Cox, K. E., Logan, C., DiCintio, M., & Thomas, C. T. (1998). Creating contexts for involvement in mathematics. *Journal of Educational Psychology, 90*(4), 730–745. https://doi.org/10.1037/0022-0663.90.4.730

Wasilewski, J. A. (2023). *The Role of Error Culture in Math Classroom Learning* [Doctoral dissertation, Northeastern University-Boston]. https://www.proquest.com/docview/27891629 3?pq-origsite=gscholar&fromopenview=true&sourcetype=Dissertations%20&%20Theses

Yıldırım, A., & Şimşek, H. (2011). *Sosyamlerde nitel araştırma yöntemleri* (8. baskı). Seçkin.

Yıldırım, İ. (2019*). Hatalı çözüm metodunun 7. sınıf öğrencilerinin bazı istatistik kavramlarındaki başarısı üzerindeki etkisi* [Yüksek lisans tezi, Adıyaman Üniversitesi-Adıyaman]. Yükseköğretim Kurulu Ulusal Tez Merkezi. http://dspace.adiyaman.edu.tr:8080/xmlui/handle/20.500.12414/2211?locale-attribute=en