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## STUDY

### DEALING WITH STUDENT MISTAKES IN MATHEMATICS AT PRIMARY SCHOOL

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#### ABSTRACT

The article focuses on how teachers deal with student mistakes during mathematics instruction at primary school. The theoretical framework encompasses a constructivist culture of teaching and learning and dialogic teaching. Two general strategies are distinguished: avoiding mistakes and using mistakes as opportunities to learn. The presented qualitative study is based on 24 audio recordings of mathematics instruction, supplemented by interviews with six teachers. Through axial coding, we arrived at seven categories that describe specific ways of dealing with mistakes: not responding; making students aware of a mistake; encouraging students to present proofs; leading students to a level where they do not fail; expanding the student network of mathematical knowledge; creating opportunities for presenting content to clarify mistakes; and providing opportunities for students to find the cause of their mistakes. The findings are discussed in the context of the constructivist approach to instruction.

#### KEYWORDS

student mistakes; dealing with mistakes; mathematics instruction; primary school; student questions

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## Introduction

Failures, errors, mistakes – all the possible ways to go wrong represent a challenge that should be managed to advance schools in the course of humanization, student orientation, and well-being. Ways of dealing with mistakes differ depending on the culture of teaching and learning that is employed in the classroom.

Traditionally, we imagine that classroom teaching and learning involve avoiding mistakes – mistakes should be concealed because the right solutions are expected in school. Teachers claim that most children are guided at home by their parents not to make mistakes (Pavelková, 2023). Some parents and their children consider a mistake to be an undesirable phenomenon. This belief about mistakes is underlined by school grades, which are often based on identifying mistakes. Mistakes can cause embarrassment, sometimes leading to situations in student learning that some authors have described as a “straitjacket” (e.g., Myhill & Warren, 2005). These situations can be overcome with learning environments that tolerate mistakes. However, this kind of learning environment is relatively rare.

Nevertheless, we can imagine – and in some schools also find – a mistake-tolerant learning environment in which failures are treated as learning opportunities. This approach involves a more thoughtful interaction of the student with the learning materials, not just aiming for the receptive absorption of information but, conversely, based on activating students, fostering mutual dialogue, and creating motivating and cognitively demanding learning tasks that involve constructive approaches to mistakes.

In this article, we work with a hypothetical continuum that stretches from the point of avoiding mistakes (which is typical for a closed, teaching-to-the-test learning culture) to the point of using mistakes as opportunities to learn (which is typical for the open, constructivist culture of teaching and learning). In other words, some ways of dealing with mistakes could be seen as *destructive* in developing student knowledge and understanding; others could be considered *constructive*.

Numerous studies have reported that the ways teachers deal with mistakes are domain specific (Oser & Spsychiger, 2005; Tulis, 2013; Tulis et al., 2017). Mistake handling is frequently investigated within the school subject of mathematics (references see below). For mathematics teaching, mistakes are often seen as stimulating and helpful (Kuřina, 2016). Most studies have focused on dealing with mistakes in mathematics at the secondary education level. However, the primary education level lays the foundation for the development of (not only) mathematical literacy, and dealing with mistakes at that level should therefore be considered very important. The presented study and its results aim to help fill the gap and enhance the theory of dealing

with mistakes in mathematics at the primary education level, an under-represented field in educational research.

This study aims to reveal various ways of dealing with mistakes during mathematics instruction at primary school on a continuum, stretching from avoiding mistakes to using mistakes as opportunities to learn. We attempt to identify teaching and learning situations in which mistakes are handled and to analyze them in the context of the culture of teaching and learning. Our research study is qualitative and employs observation and interview as research methods. We hope our findings will be relevant for researchers as well as for practicing teachers. For illustration, we use authentic extracts from lessons and interviews with teachers that enable us to discuss the issue of dealing with mistakes in relation to the quality of instruction in the context of the changing culture of teaching and learning.

## 1 Theoretical background

According to Tulis et al. (2016), *errors* are defined as an

unintended discrepancy between a current and desired state, or as a deviation from a given standard ... Failure implies more than just this perceived discrepancy. In contrast to errors, failure experiences constitute a more global miss of a goal with a greater focus on the subsequent consequences (p. 15).

In the context of teaching and learning, mistakes provide references to knowledge gaps or misconceptions (Grassinger et al., 2018) and thus have a high potential to support learning.

Errors (mistakes) are often studied in the context of learning. From a psychological perspective, “learning from errors is conceptualized as self-regulated learning in the specific situation where an error becomes salient to an individual” (Grassinger et al., 2018, p. 38). However, in our study, we focused on “learning from errors in the classroom,” which implies that we do not emphasize student self-regulation to such an extent; we stress the productive classroom discourse in which teachers handle mistakes – professionally – to support learning among students.

### *1.1 Mistakes and the culture of teaching and learning*

There is a link between dealing with mistakes and the culture of teaching and learning. *Learning culture* refers to a “time-bound complex of certain forms of learning and teaching styles and related anthropological, psychological, social and pedagogical orientations” (Weinert, 1997, p. 12). Time-bound refers to the changes that the culture of teaching and learning goes through. The “new” (constructivist, productive) culture is sometimes contrasted with

the old (receptive, reproductive) culture of school teaching and learning, which has become outdated and stagnant, so it is necessary to reject it and start anew. However, such simplified views (either – or / from – to) are not entirely fitting and are balanced by texts that grasp the problem of the changing culture of teaching and learning in a more nuanced way in the sense of not only – but also (Reusser, 2001).

There is not a definite set of characteristics of the “new” (productive) culture; however, proposed qualities have included activity and (cognitive) activation, constructive dealing with mistakes, cognitive heterogeneity, and generative, rather than algorithmic, problem solving (Reusser, 2006). Other authors have listed characteristics such as individualization of learning processes, cognitive activation of students, the introduction of authentic learning tasks requiring the transfer of learned content to new contexts, generative problem solving, verbalization of problem solving, and support of metacognitive processes (Janík, 2013; Reusser, 2001; Weinert, 1997; Wiater, 2005).

For the constructivist culture of teaching and learning, the shift from formalisms to natural processes of cognition and learning is significant (Hejný, 2012). However, these processes are idiosyncratic and, to a certain degree, visible. For the culture of teaching and learning, this means putting more emphasis on differentiation, individualization, and explicitness of students’ learning processes. *Making learning visible* is one of the slogans of constructivist teaching and learning culture. This means that student mistakes will also become more visible, which can be challenging for students, especially when the mistake climate in the classroom is not positive (Ingram et al., 2015; Steuer & Dresel, 2015) and when the way mistakes are addressed is not sensitive enough. However, explicit work with mistakes is vital for learning at the individual level and at the whole-classroom level. As Brodie (2014) pointed out, students should be confronted with mistakes as they are seen as evidence of their thinking and needs, while teachers can draw on students’ reasoning underlying their mistakes to help them develop mathematical concepts. Public analysis and discussion about mistakes could support conceptual understanding.

In sum, constructive dealing with mistakes is possible in a mistake-tolerant learning environment. When mistakes occur, they are used here as opportunities to learn. On the contrary, for a mistake-hostile environment, the strategy of avoiding mistakes is typical. However, this strategy is restrictive in terms of supporting student learning. Avoiding mistakes results in the fragmentation of content to be learned, narrowing the selection of learning tasks, and reducing students’ cognitive demands and cognitive activation. The complexity and durability of knowledge are, therefore, also limited. It means that tackling ways of dealing with mistakes in the classroom is crucial.

### *1.2 Ways of dealing with mistakes*

Based on the literature, two distinct ways of dealing with mistakes could be distinguished: (1) avoiding mistakes and (2) using mistakes as opportunities to learn (Ingram et al., 2015; Pan et al., 2020). The authors considered the shift from the strategy of “avoiding mistakes” toward the strategy of “using mistakes as opportunities to learn.” However, it is not easy to achieve this shift because both strategies are closely related to the teacher’s beliefs in a broad sense (including beliefs about the teaching and learning of mathematics – Voss et al., 2013); these beliefs guide their behavior in the classroom.

For many reasons, the strategy of “avoiding mistakes” is widely used in practice (see Heinze, 2005; Meyer et al., 2006 for Germany; Schleppenbach et al., 2007 for the United States). One reason teachers use the strategy of “avoiding mistakes” during the lesson is that mistakes become more widespread and are remembered (Swan, 2002). Another reason for avoiding mistakes is that students often find that mistakes are face-threatening and accompanied by negative feelings (Käfer et al., 2019; Zander et al., 2014). If the teacher’s beliefs are transmissive, they believe that a mistake can have a counterproductive effect on learning, and they may thus employ error avoidance didactics (Köpfer, 2022).

The strategy of “using mistakes as learning opportunities” could be challenging for teachers as well as for students. In principle, it is based on generating cognitive conflicts in the classroom. It means mistakes resulting from misconceptions are welcomed in the classroom, and they are confronted with each other and discussed in terms of the content to be learned. Mistakes serve as boundary markers (Grassinger et al., 2018), distinguishing more appropriate from less appropriate views of the concept and more acceptable from less acceptable practices in the field. Here, teaching and learning are seen as enculturation into a community of practice (e.g., doing mathematics, acting as mathematicians).

This approach aligns with the socio-constructivist approach in didactics (Janík et al., 2024). It requires that students are ready to participate in teaching and learning with a high level of (cognitive) activation. Teachers must have solid content knowledge and pedagogical content knowledge, including three specific error-handling competencies: knowledge about domain-specific errors by students, knowledge about strategies for handling errors, and belief in the potential benefits of student errors (Wuttke & Seifried, 2017). Such knowledge and competencies enable the teacher to engage in dialogic teaching, which seems promising in enabling constructive dealing with mistakes (Šedřová et al., 2020).

### 1.3 Dialogic teaching and student questioning

According to Šedřová et al. (2020) dialogic teaching refers to a style of teaching that harnesses talk to engage student interest, stimulate their thinking, advance their understanding, and at the same time empower them. It involves students engaging in communication and their willingness to respond to the teacher's questions and actively ask the teacher questions. Student questioning – open and divergent – is how meanings are negotiated in the classroom (Janík et al., 2020).

Student questions in the classroom allow them to take initiative and responsibility for their learning. Mistakes that students make during a learning task are a natural part of learning. However, past research presented ambiguous findings on the role of mistakes in student learning (Rach et al., 2013). Kuřina (2016) summarized this issue for teaching and learning mathematics with two approaches. One is based on knowledge transmission (in Czech: *pouhé předávání poznatků*), where student questions and mistakes are not welcomed; the second is based on constructive cognition (in Czech: *přirozený poznávací proces*), where student questions and mistakes are essential.

In the second approach, mistakes are spontaneously addressed in the classroom and used as learning opportunities. The strategy of public questioning – identifying, analyzing, and discussing mistakes – is believed to support conceptual understanding. Within dialogic teaching, the teacher creates an environment based on discussions generated by student questions, in which students communicate how they think about the presented learning material and possible solutions. Mistakes may be present in these questions, revealing the student's current understanding and misconceptions about the mathematical concepts used.

Teacher responses to mistakes displayed in student questions have significant learning potential, especially when the teacher asks students to present, compare, confront, justify, and evaluate their answers or strategies. Teachers may also ask students to determine the validity of their answers (Pinzón et al., 2022). Within dialogic teaching, various types of questions were developed (e.g., Drageset, 2019) to deal with mistakes constructively, aiming to make the thinking and understanding of the student visible not only to that student but also to others in the classroom. The teacher is present not to correct mistakes but rather to create opportunities for students to find their own mistakes based on the interaction with learning material and discussion with other people (or information resources) in the classroom or outside. In this way, the clarity, structure, and coherence of the content to be learned increases, which makes the content more accessible for students (Janík et al., 2024). With this understanding, we approach student mistakes in our study presented in this article.

### *1.4 State of current research*

Mistakes in the classroom have held the attention of researchers for decades. The research field is broad and diverse. Regarding the topic of our article, two research areas are especially relevant: research on dealing with mistakes in the (mathematics and physics) classroom (Borasi, 1994; Heinze, 2005; Meyer et al., 2006) including the focus on cultural variability (Matteucci et al., 2015; Santagata, 2004, 2005; Schleppenbach et al., 2007), and research on the role of mistakes (on making mistakes and treating mistakes, on mistake culture or error climate) with regard to other variables as components of the learning environment within the respective culture of teaching and learning: student motivation (Rakoczy et al., 2008), student achievement (Steuer & Dresel, 2015), resp. academic achievement (Grassinger et al., 2018), and others. Our focus is on the area of research on dealing with mistakes in the classroom.

In a qualitative study, Borasi (1994) explored how errors could be used as “springboards for inquiry” in mathematics instruction. She conceptualized errors as learning opportunities and suggested that the following activities could be explored utilizing errors: experience constructive doubt and conflict regarding mathematical issues, engage in challenging mathematical problem solving, pursue mathematical explorations, reflect on the nature of mathematics, experience the need for monitoring and justifying their mathematical work, experience initiative and ownership in their learning of mathematics, recognize the more humanistic aspects of mathematics, and verbalize their mathematical ideas and communicate them.

Heinze (2005) conducted a video study focusing on approaches to dealing with mistakes in mathematics (geometry) lessons at the secondary level in Germany. The video study encompassed 22 lessons of geometry instruction in the eighth grade of secondary schools. This involved eight classes across four secondary schools, with two to four consecutive hours devoted to the topic of proof and reasoning in geometry in each class. Within the recorded lessons, a total of 104 situations containing mistakes were identified. They varied in type: mistakes in the expression of the content (25%), factual mistakes (23.1%), methodological mistakes (8.7%), and mistakes in logical argumentation (43.3%). The higher incidence of mistakes in logical argumentation is attributed to the fact that the analyzed lessons focused on proofs and reasoning in geometry, where argumentation is a vital part of the curriculum. Teachers identified mistakes in 84.6% of cases, classmates in 15.4%. In 87.5% of cases, teachers responded to the mistakes; in 9.6% of cases, classmates responded; and in 2.9% of cases, students who made the mistakes responded. Of the mistakes, 9.6% received no reaction, 25% were corrected by stating the correct answer, 17.3% were addressed with an explanation, and 48.1% posed a challenge for further work in the lesson.



Discussing the research findings, the author points out the issues of deductive questioning-based teaching (*fragend-entwikelnd*), which, strongly guided by the teacher, allows relatively little room for student mistakes. The author considered the average of 4.72 mistakes per lesson to be low, given that mistakes are an integral part of the learning process.

The Video Study Physics focused, among others, on dealing with mistakes in the classroom (Meyer et al., 2006). Based on an analysis of 100 video recordings of Physics lessons (two ninth-grade lessons in fifty – Gymnasium and Realschule – during the 2003/2004 school year), Meyer et al. (2006) showed the following picture:

In all classes occurred situations where students displayed anxious and/or shameful behavior. Only in six lessons from 100 were situations for the raters evident where students displayed anxious behaviour. ... However, different distribution was found regarding “avoiding mistakes” and “negative responses to mistakes” ... in many classes, these aspects were not present ... explicit mistake-situations and negative responses to them were rare and not intensive (pp. 31–32).

Another problem the IPN Video Study Physics revealed was confusion between learning-oriented situations (where informative feedback is needed) and performance-oriented situations (where students are provided with evaluative feedback). Learning situations emphasize acquiring knowledge; performance-oriented situations focus on succeeding and avoiding mistakes. Confusing these two types of situations seems undesirable because it can reduce motivation to learn and can affect the quality of instruction.

Santagata (2004, 2005) is a leading researcher presenting findings related to cultural variation within teacher beliefs about mistakes and related practices. Based on observational methods (including video studies), research has revealed the following characteristics – as Pan et al. (2020) summarized:

American teachers tend to minimise or deemphasise students’ errors, Italian teachers tend to be overtly critical of errors, and Japanese and Chinese teachers often have a positive attitude towards errors and devote substantial amounts of time to discussing them with their students (p. 1106).

Matteucci et al. (2015) investigated teacher beliefs about the role of errors in learning and the strategies used in classrooms to handle student errors. The authors used the Error Orientation Questionnaire (Rybowiak et al., 1999) to capture teacher beliefs and a video study with three Math teachers to analyze error-handling strategies. Using cluster analysis, two opposite error orientations – negative vs. positive – were found. A subsequent exploratory analysis based on video data showed crucial results. Matteucci et al. (2015) stated:

... teachers with positive error orientation used more positive strategies and had higher levels of mastery achievement goal, self-efficacy, and mastery learning practices. ... [they] perceive also high strain in front of errors. ... [they] do not use only better educational practices, but they also show students how to handle errors in an efficient way (pp. 1–2).

Classroom research has highlighted a variety of ways of dealing with mistakes during teaching and learning (mathematics). In line with theoretical considerations, the research revealed two main strategies: avoiding mistakes and using mistakes as opportunities to learn. In research studies, both strategies are differentiated and examined in relation to other variables. How these strategies are used in practice and how they relate to the changing culture of teaching and learning is still an open question that we aim to approach in the present study.

### *1.5 The present study*

Dealing with mistakes is often researched in mathematics classrooms, but usually not in primary schools. Due to the limited knowledge about how teachers deal with mistakes in student questions, we conducted a qualitative research study on mathematics instruction at primary schools in the Zlín region (Czech Republic). The main research question was: How do teachers deal with students' revealed mistakes and how does this serve as a starting point for subsequent learning?

## **2 Method**

### *2.1 Sample and data collection*

We used data collected primarily for the PhD thesis of the third author of this article (Pavelková, 2023) to answer our research question. We analyzed 24 audio recordings of mathematics lessons taught by six female primary teachers (Table 1). We did not ask the teachers to teach the same content because they worked in different grades. The teachers decided which lessons would be observed and audio recorded. All teachers were also interviewed. A semi-structured interview followed immediately (or as soon as possible) after the observed lesson. The length of the individual interview was 20 to 40 minutes. Several questions were prepared regarding the lesson's structure with a focus on student activity, where students were confronted with mistakes and had the opportunity to ask questions.

As a qualitative research design was planned, teachers who met the following criteria were selected:

1. The teachers with teaching qualifications as primary teachers.
2. Teachers who wanted to participate in the research and consented to data collection in the mathematics lessons.
3. In the course of their teaching, communication takes place and mistakes appear.

Table 1

*Description of the research sample (the names of the teachers were changed)*

Teachers	Years of experience	Grades	Numbers of students	Number of recorded lessons/duration in minutes
Jana	20	3 <sup>rd</sup>	18	4/180
Eva	3	2 <sup>nd</sup>	17	1/45
Veronika	6	1 <sup>st</sup>	9	10/480
Radka	7	4 <sup>th</sup>	20	2/90
Zuzana	30	3 <sup>rd</sup>	16	5/270
Iva	2	1 <sup>st</sup>	18	2/90

## 2.2 Analyses

Audio-recorded data from mathematics lessons were transcribed and supplemented with notes from observations carried out by the third author of this article during data collection between 2017 and 2019. Codes related to dealing with mistakes were developed based on axial coding. The codes covered primarily communication sequences that described the mutual interaction of participants; they were mostly student questions. In the next step, seven categories were created from the codes (Table 2).

Data from interviews with teachers were transcribed and processed by open coding. The developed codes supplemented the categories created from the audio data.

## 2.3 Results

Seven categories were created from the analyzed data (Table 2). They enable us to answer the question: What are the ways of dealing with mistakes in mathematics instruction at primary school?

In line with our theoretical background (from “avoiding mistakes” to “using mistakes as learning opportunities”), these categories stretch on a continuum from the point where the teacher does not react to the mistake (overlooking) to the point where the student determines how to deal with the mistake.

The left column designates situations that emerge from the data, and the right column lists “didactic tools” for dealing with mistakes.

Table 2

*Teacher ways of dealing with mistakes and related didactic tools*

	<b>Ways of dealing with mistakes</b>	<b>Didactic tools for dealing with mistakes</b>
1.	not responding to a mistake	without reaction, “overlooking”
2.	making students aware of a mistake in their solution or argument	open questions from the teacher
3.	encouraging students to present proof of their solution to challenge mistakes	different types of questions; using the student’s language
4.	leading students to the level where they do not fail	graded tasks
5.	expanding the student network of mathematical knowledge concerning mistakes	rhetorical questions; description of the related content
6.	creating opportunities for different ways to present the content to clarify mistakes	illustrations, play/drama, didactic tools and media
7.	providing students with opportunities to search for the cause of mistakes by themselves	adaptive scaffolding

Transcripts from audio recordings illustrate the results (T = teacher, SX = student, and X = the student’s initial letter).

### 2.3.1 Teacher does not respond to a mistake

Teachers are well aware of which students make mistakes. However, some teachers will not address mistakes during teaching. As explained by teachers in interviews, the main reason for mistakes made by students in mathematics is the pressure for speed. “Kids make mistakes because of rushing. It bothers me that when they ask, they ask with such mistakes that I have to go back to the basics. It is just a circle.” Teachers admit that they do not respond to student mistakes and expect students to “learn somehow,” which they attribute to the maturing of students or to the expectation that “students will understand it later.”

**Example 1:** Within a teaching unit on numerical sequences, the teacher deepens arithmetic knowledge among fourth-graders and focuses on understanding the decimal system and supporting logical thinking. The teacher incorporates the content to be learned into creating several possible numerical sequences and asks the student to find one of the possible solutions.

T: Try to complete this numerical sequence: 99, 88, 77, ... 98, 87, 76, ...

SN: I do not understand.

ST: It is easy: 87, 76, 65... and so on. Is it correct?

SN: How come?

T: (*The teacher walks among the students and assigns another task for those with no severe problems with the numerical sequence.*) Create a task for us and come up with your own numerical sequence.

Example 1 shows that the teacher does not respond to the student who is unsure of what to do. However, another student who considers the assigned task to be easy does respond. This prompts the first student to engage again, seeking assistance in the form of a question. The teacher moves around the classroom, and instead of assisting the students, she assigns another task – a new one.

### 2.3.2 Teacher makes students aware of mistakes

Teachers mention that they have no qualms about making a mistake and waiting for the student's response, or they ask students provocative questions about their mistakes: "I often do that, and I like it, then they (introverted children) react, too. They discuss it and argue. They run from desk to desk and compare their results. They ask me; they ask each other" (Jana). Considering the correctness of solutions and thinking about various unusual problem-solving strategies influences student thinking in general, not only in mathematics. When a teacher responds to student mistakes in the classroom with a stimulating question, it encourages more demanding mental operations in other students. "Many times, when they saw others, when I asked about one student's mistake, it inspired others to come up with a completely new solution" (Jana).

**Example 2:** The example is from an arithmetic class for third-graders, in which the teacher aimed to seek and sort data and create graphs or tables. The teacher's goal was to create opportunities to recognize the mutual connections when using length, weight, or parts units. The lesson's topic was recording decimal numbers and their use in time units.

SP: Why do you calculate this? Can't you write 1.5? (*The student takes a blank sheet of paper, starts converting minutes into hours, and finishes his task. He goes to the desk of a classmate.*)

SL: 1.5 of what?

SP: It does not work; I want it to be exact.

T: Can we write it as a decimal, children?

SP: Yes, you can, but it is not exact.

SV: Why not? What is the difference between three and a half hours and 1.5?

The students tried to convert time units into decimal numbers. One student had a proposed solution, but this was not exact, and he did not know how to make the recording accurate. Instead of helping, the teacher asked other students if it was possible to write it as a decimal number – it seems the teacher wanted to confirm with others that it was possible. A student who wanted an accurate recording but probably did not know how to achieve it entered the interaction again. The teacher again did not react; another student did, offering his way of helping.

### 2.3.3 Teacher encourages students to present proof of their solution to challenge mistakes

Teachers provide students with the opportunity to come up with their solutions. The correctness of the result, or even the mistake, serves as a stimulus for the student to present their problem-solving approach to others. Furthermore, students should identify the mistake by themselves if the result is incorrect. As teacher Zuzana explained in the interview: “Children look for the mistake, and when they find it, they slap themselves on the forehead.” Learning mathematics is encouraged by an environment open for presenting mathematical proofs, as mentioned by teacher Radka in the interview: “They learn from mistakes because when they make a mistake, they are forced to solve the problem again. I always tell them that they are good when they discover a mistake, and we even praise them, saying that we have our first detective who uncovered it... I also tell them that every detective should be able to describe why it happened. For example, when robbers rob the bank, I always ask, how did they get into the house? Describe it to me as well.”

**Example 3:** In the second grade, the lesson topic was axial symmetry. The teacher asked the students to find how many axes of symmetry the given traffic signs have.

SL: Is there a sign that does not have any axis of symmetry?

T: Try it.

SP: I do not know, not at all.

SK: Yes, I found this sign here and folded it in half, and that is it. (*Throws away the paper on which he was drawing and walks away.*)

T: Do you really think it is the circular one? (*The teacher points out that it is indeed a circular sign, but inside, there is an arrow pointing right, which is not axially symmetrical.*)

SL: But here it is, isn't it?

T: Kájo (student SK), will you finish it? (*Encourages the leaving student who folds the cut-out sign.*)

T: At what shape would you find the axis of symmetry?

SL: He probably does not want to continue anymore.

SK: (*With tears, he crosses out the entire diagram and leaves.*)

T: It is okay.

SK: What about this letter you mentioned? (*Speaks to the teacher.*)

T: Symmetrical, you mean?

SK: Do letters also have axes? What about the globe? I have seen that. What about a butterfly? Also?

Student K sketched an axis of symmetry on a circular sign, but for the picture inside the sign – an arrow to the right – the axial symmetry was not applicable. Figuring out the mistake by the teacher caused an irritated reaction from the student, resulting in the entire solution being crossed out. The student felt the presence of a mistake in his solution, but he could not localize it. After a while, the student re-entered the dialogue with the teacher and asked about symmetry in letters and other things.

### 2.3.4 Teacher leads students to a level where they do not fail

In their interviews, teachers described the presence of student mistakes as mirrors of the lack of understanding of the content that was explained earlier. Understanding the learning task depends on understanding the mathematical context and methods. For example, Zuzana stated that she did not understand “when the task is solved at the board, why mistakes are made at the desk. I tried to change the workbook, but it probably will not help. In the end, I gave them those red graduated cards. That worked for me.”

Another teacher (Iva) stated that mathematics is not just about “reciting mathematical formulas;” she therefore tried to incorporate “graded tasks, where children are not afraid to tackle more difficult problems. However, some children simply do not understand it yet and want to experience success, so they choose a simpler option” that ensures understanding of the content and thus also the advancement of the students.

Teachers aim to stimulate student thinking and their desire to search and explore based on their individuality. It is evident from the observations that graded tasks are not only incorporated by teachers in connection with prepared levels; teachers also transform the principle of task grading where one common correct solution would be expected.

**Example 4:** The teacher commented on the presented task, in which second-graders competed in paper collection, and assigned a task for the students to propose a similar graph. She left it up to them because she said that what is put in the graph and what type of graph is used was up to each student.

SP: How much fruit do I eat in a day?

T: Why did you choose fruit? What exactly interests you? (*Points to the graph in the picture in the textbook.*)

SP: Because those are kilograms, and I can do that.

SL: Which subject takes up the most of my study time? (*The student has a graph sketched and comes to the teacher.*)

SL: What are all the school subjects that exist?

SH: What else are we learning?

T: Do you mean subjects? (*Points to the class schedule.*)

SL: How do I write it in the graph if I read twice a day?

T: What units will the graph be in?

SZ: Shouldn't you write it in hours?

SL: But I do not even read for an hour; that is too much (*laughter*).

The students asked the teacher questions to clarify whether they chose the right area to represent with a graph. The teacher left it up to each student to choose anything they could do. One student works with kilograms and commented that he could do this. Another student wanted to write that he reads twice a day, which is a more complex task than working with kilograms.

### 2.3.5 Teacher expands student network of mathematical knowledge concerning mistakes

Teachers know that solving a word task sometimes requires knowledge from multiple thematic parts of the curriculum. Thus, students may make mistakes due to an inadequate understanding of previously learned content. Teachers aim to create an environment in which students do not memorize mathematical content but are instead assisted with questions related to the content. This approach helps build a knowledge structure that lays solid mathematical foundations for subsequent learning. Sometimes, it may seem like teachers advise students during teaching – as teacher Eva explains in the interview: “When there is difficult content, I ask questions like whether they need, for example, a square grid or a multiplication table or other things.”

**Example 5:** In the second grade, the content concerned arithmetic. The teacher and students were solving the following word task: There are 25 apple trees in the garden. There are 14 apple trees in the park. How many apple trees are in the park and garden altogether?

PL: Where should I write it?

T: In your notebook.

PK: Is it like this?

PR: How many apple trees?



T: Read the task. Do you know how to solve it? What would help you? Maybe pegs? Or to know which mathematical operations we are already able to use?

Reaction of students: No, yes, yes, no...

T: Who does not know? Here is the interactive board; you can help yourselves.

T: Leni, what don't you understand?

PL: I do not understand it. How should I solve it? (*The teacher leans towards the girl and asks.*)

T: Well, what don't you understand?

PL: I do not understand. How should I do it?

T: Would a number axis help you?

The teacher set up a situation that the students had experienced several times, but word tasks are cognitively more demanding, so some students still needed help. The teacher instructed them step by step and then asked an assisting question. Because she noticed that not all of the students were working, she began to offer aids to help, and then she started to remind the students of mathematical operations and instruments (the number axis).

### 2.3.6 Teacher creates opportunities for different ways of presenting the content to clarify mistakes

In the interviews, teachers pointed out that students do not always understand the content as the teacher explains it. Therefore, they must find various ways to allow the student to discover mathematics in a familiar environment and challenge their own experiences (with their authentic mathematical knowledge). For first graders, this could be the possibility of counting on their fingers. As the teacher Zuzana pointed out in the interview: "We all have them with us... But there is also an abacus there, or beads in a bowl. They can choose the cubes. Sometimes, I have noticed that a child who is lazy and does not go for anything does not produce the solution" (Zuzana).

**Example 6:** The lesson's topic is arithmetic in the third grade and is concerned with the relationship between two phenomena.

PS: How can I write three pieces here?

T: What do you mean?

PT: We have like three pieces here. Don't you mean three parts?

PS: I do not mean that, I mean three pieces 1, 2, 3. How do I write three pieces?

T: Three pieces of what?

PS: Three pieces of oranges. I need to write three pieces, not a kilogram.

T: Would weight help? (This is followed by a demonstration where the student weighs apples on a kitchen scale, and the teacher calls other children to see how much the apples weigh.)

T: Does anyone know what “g” means?

S: Grams.

The student was put in a situation that he could not solve. He explained his ideas to the teacher, who offered help in the form of a kitchen scale, showing the weight. However, since the weight did not correspond to exactly one, two, or three kilograms, the teacher introduced “grams” as another unit of weight that could help solve the task.

### 2.3.7 Teacher provides students with opportunities to search for the cause of mistakes by themselves

As the teachers pointed out in the interviews, some students prefer to choose their own pace and want to figure out the mistake by themselves, even at the cost of increased time investments. Sometimes, other students join, and “when one makes a mistake, and they have to find where, that is like wow, they figured it out themselves. And I wonder how they engage in posing questions to each other. One after another, asking why and why... The time is limited, but it is nice to see how mistakes trigger learning” (Veronika). These students were internally motivated to understand the content. At the same time, self-regulated learning was fostered.

**Example 7:** The lesson presents an interesting task (created by one of the students) based on sorting words (cities).

SL: Can you complete my descending sequence? Task: Olomouc, Vsetín, Žatec, Brno...

SF: Is it descending?

T: Yes, descending, although I am not sure myself. (*Laughter*)

SK: I know, is it about which letter it starts with? O, V, Ž ... And why is there a B?

SL: Nooo.

T: Would it help someone to repeat the alphabet? Or what would help you? Do you want help?

SK: Lucka, do you know a city with three...? (*The teacher approaches student SK.*)

SP: Is it that there is one less letter in each word?

T: What would you add there?

SL: Nooo, I am not good at this; I do not know cities... I do not want to make it up.

T: Luci, do you have a descending one?

SH: Just guess why it is like that.

SP: Don't you know? Is it a descending sequence, or is it going up? Could it be like this?

The teacher first specified the task in a descending sequence. The students asked each other questions while the teacher intervened with suggestions of what could help them. After the students did not react, as they were still discussing it amongst themselves, the teacher asked if they wanted any help. The students, again, did not respond to the teacher and continued searching for a solution amongst themselves.

### 3 Discussion and limitations

The ambition of the research study presented in this article was to delve into ways of dealing with mistakes in mathematics classrooms in primary school. Our research revealed seven basic responses across multiple observed mathematics lessons. The primary direction for handling mistakes in the classroom is the intentionality of mistake occurrence: teachers either intentionally prepare and incorporate mistakes into the lesson, or the mistakes naturally emerge during the lesson. In the latter case, it was up to the teacher to decide whether to “overlook” the mistake or to build on it in subsequent learning.

These seven responses, situated on a hypothetical continuum from avoiding mistakes to using mistakes as learning opportunities, indicate the degree of student autonomy in working with mistakes in the classroom. Our findings align with those of Ingram et al. (2015) and Pan et al. (2020). At one end of this continuum, handling mistakes is highly teacher-directed, meaning the teacher decides whether to work with mistakes or not. At the other end of the continuum, students have a high degree of autonomy, deciding how, when, for how long, and with whom they will work with mistakes, while the teacher provides them with opportunities for autonomous mistake processing. Students take control over their activity within the learning process; elements of self-regulation occur. Teachers act as facilitators.

In situations highly directed by the teacher, as demonstrated by many authors (Heinze, 2005; Meyer et al., 2006; Schleppenbach et al., 2007), it was found that the teacher tends to overlook the mistake, meaning that the teacher is aware of the mistake but decides not to address it further for various reasons. A common reason – as revealed in our interviews – was that teachers had already devoted so much time to the content that they did not want to spend more time on it. Some teachers added that students would understand the content later. Teachers were aware that each child had their own individual

pace. However, the teachers did not want to respect students' pace, especially in situations where they believed the student should have already mastered the content. Another issue regarding pacing is that children try to be fast and compete with each other, and as a result, numerous mistakes arise that would not occur at a slower pace. Some teachers intentionally avoided mistakes so that others would not remember it, which aligns with findings by Swan (2002) and others.

Some authors agree that it is desirable to make mistakes visible within teaching and learning (Brodie, 2014) because dealing with mistakes is a natural process that provides evidence of student thinking and reveals student needs in developing mathematical concepts and understanding. Further categories that emerged in our research are relevant to this premise. The fastest way to acknowledge mistakes in teaching and learning involves teacher alertness to the occurrence of mistakes. Alerts typically came through various questions, although some teachers would alert students through declarative or imperative instructions (e.g., "Watch out, try it again"). This refers to a possible shift in the culture of teaching and learning that is no longer strictly transmissive but not entirely constructivist either. We can understand this phenomenon as a shift from formalisms to natural processes of cognition in teaching and learning.

Another emerging category refers to encouraging students to present proof of their solutions. Mathematics is built on proofs and proving. With younger students, the expectation of being able to provide proofs for mathematical discoveries, argue, or even find mistakes in pseudo-proofs is often not present (Pavelková, 2023). However, providing proof, determining correctness, and arriving at results could reveal student knowledge gaps or misconceptions (Grassinger et al., 2018), which have great potential to support learning. Teachers reflect the need to bring students to a level where they have a solid foundation and do not make mistakes. To achieve this, teachers used so-called graduated tasks to practice specific content. These tasks are divided into three or more difficulty levels, and students choose a level corresponding to their current, eventually proximal knowledge level. This is the next category that emerged from our research. Another strategy teachers employed after the mistakes occurred was expanding the mathematical knowledge structure in students' minds; that is, teachers attempted to create connections between topics so that students perceived content units as interconnected rather than isolated. Teachers also recognized that students used various learning strategies. Therefore, when a student made a mistake, the teachers tried to respond by presenting the content to be learned, for example, through dramatization or using additional visual aids.

Besides identifying the seven categories of dealing with mistakes, we found that teachers and students utilize questions. Within the strategy "using mistakes as learning opportunities," teachers engaged in dialogue based on

student questions when mistakes occurred in the classroom. These were often open-ended questions that encouraged students to formulate their own solutions, which revealed their thinking processes. To a lesser extent, teachers used closed-ended questions, to which students typically responded with single-word answers. Teachers also used rhetorical questions to prompt students to think about their solutions or results, remind them of connections to other topics, or alert them to mistakes in their results. Another tool for working with mistakes was utilizing the language used by the students. Teachers gave students space for mutual discussion and questioning, enabling them to identify where the mistake occurred or communicate their problem-solving strategies and results. In some cases, teachers admitted that they did not understand student thinking processes and the ways students operated with the content. Although there is still a tendency to prevent students from making mistakes (Heinze & Reiss, 2007), for students, it is pretty natural to be confronted with their own mistakes and to observe others making mistakes (proxy mistakes). This way, students receive feedback on their own mistakes or lack thereof. In any case, active engagement with mistakes leads to learning with understanding and a significant development in student understanding of the content. Another tool used in working with mistakes was teachers' efforts to present or illustrate the same content differently (c.f. Janík et al., 2020). They utilized various didactic aids, including graduated tasks, visualization, and dramatization.

Our research study was conducted in mathematics classes. Since we did not observe teachers in other school subjects, it cannot be stated whether the teacher responses are mathematics-specific or whether teachers across all subjects would use the same responses. Furthermore, Majčík (2022, p. 75) pointed out that “the manner of productive work with mistakes may also vary depending on the goal of the learning task, which influences the nature of questions that teachers use.” These are some of the limitations of our research study; however, they can be seen as prospects for further research.

## Conclusion

The presented study focused on teacher responses to student mistakes. Based on qualitative data analysis, it was found that teachers recognize situations when students make mistakes. However, they decide whether and how to address classroom mistakes. The findings from our research investigated ways in which teachers and students deal with mistakes in the classroom. We could show that teachers strive to address mistakes in the classroom. However, we can infer that they need to expand their repertoire of mistake-handling strategies. Based on our analysis of dialogues between teachers and students

in the classroom, teachers need to be more aware of the content of instructional communication toward curriculum goals.

In our study, we examined teachers who may have found themselves in the transition from an instructivist (behavioral) approach, where mistakes are seen as signs of inadequate mastery of the content, toward a constructivist approach, where mistakes are seen as indicators of the state of student learning (Gagatsis & Kyriakides, 2000) and used as opportunities to learn. However, it takes work to utilize mistakes in such a constructive manner. The prerequisite for this is a high level of teacher pedagogical content knowledge, including familiarity with tools for addressing mistakes sensitively in terms of content and students. In addition to that, Wuttke and Seifriedet (2017) emphasized the importance of teacher belief in the potential benefits of student mistakes.

Mistake-related beliefs refer to teacher views on failures and their role in teaching and learning. Because beliefs act as cognitions that guide teacher practices, they should be cultivated positively when the aim is to utilize mistakes as opportunities to learn. It is an “irony of fate” that mistakes are predominantly seen as negative in schools, which results in avoiding them within teaching and learning. The hidden potential of mistakes as opportunities to learn should, therefore, be (re)discovered from time to time and developed step by step.

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